Lecture 8
2018/2019

Microwave Devices and Circuits for Radiocommunications

2018/2019

- 2C/1L, **MDCR**
- **Attendance at minimum 7 sessions (course + laboratory)**
- Lectures- **associate professor Radu Damian**
  - Friday 09-11, II.13
  - E – 50% final grade
  - **problems** + (2p atten. lect.) + (**3 tests**) + (bonus activity)
    - 3p=+0.5p
  - **all materials/equipments authorized**
- Laboratory – **associate professor Radu Damian**
  - Wednesday 12-14, II.12 odd weeks
  - L – 25% final grade
  - P – 25% final grade

Materials

- http://rf-opto.etti.tuiasi.ro

Laboratorul de Microunde si Opt ×

Not secure | rf-opto.etti.tuiasi.ro/microwave_cd.php?chg_lang=0

Main **Courses** Master Staff Research Students Admin

Microwave CD Optical Communications Optoelectronics Internet Antennas Practica Networks Educational software

Microwave Devices and Circuits for Radiocommunications (English)

Course: MDCR (2017-2018)

Course Coordinator: Assoc.P. Dr. Radu-Florin Damian
Code: EDOS412T
Discipline Type: DOS; Alternative, Specialty
Credits: 4
Enrollment Year: 4, **Sem.** 7

Activities

Course: Instructor: Assoc.P. Dr. Radu-Florin Damian, 2 Hours/Week, Specialization Section, Timetable:
Laboratory: Instructor: Assoc.P. Dr. Radu-Florin Damian, 1 Hours/Week, Group, Timetable:

Evaluation

Type: **Examen**

A: 50%, (Test/Colloquium)
B: 25%, (Seminary/Laboratory/Project Activity)
D: 25%, (Homework/Specialty papers)

Grades

Aggregate Results

Attendance

Course
Laboratory

Lists

Bonus-uri acumulate (final)
Studenti care nu pot intra in examen

Materials

Course Slides

MDCR Lecture 1 (pdf, 5.43 MB, en,)
MDCR Lecture 2 (pdf, 3.67 MB, en,)
MDCR Lecture 3 (pdf, 4.76 MB, en,)
MDCR Lecture 4 (pdf, 5.58 MB, en,)

Examen: Logarithmic scales

dB = $10 \cdot \log_{10} (P_2 / P_1)$		dBm = $10 \cdot \log_{10} (P / 1\ mW)$	
0 dB	= 1	0 dBm	= 1 mW
+ 0.1 dB	= 1.023 (+2.3%)	3 dBm	= 2 mW
+ 3 dB	= 2	5 dBm	= 3 mW
+ 5 dB	= 3	10 dBm	= 10 mW
+ 10 dB	= 10	20 dBm	= 100 mW
-3 dB	= 0.5	-3 dBm	= 0.5 mW
-10 dB	= 0.1	-10 dBm	= 100 μW
-20 dB	= 0.01	-30 dBm	= 1 μW
-30 dB	= 0.001	-60 dBm	= 1 nW

[dBm] + [dB] = [dBm]

[dBm/Hz] + [dB] = [dBm/Hz]

[x] + [dB] = [x]

Examen

- Complex numbers arithmetic**!!!!**
- $z = a + j \cdot b$; $\mathbf{j^2 = -1}$

Impedance Matching

Matching , from the point of view of power transmission

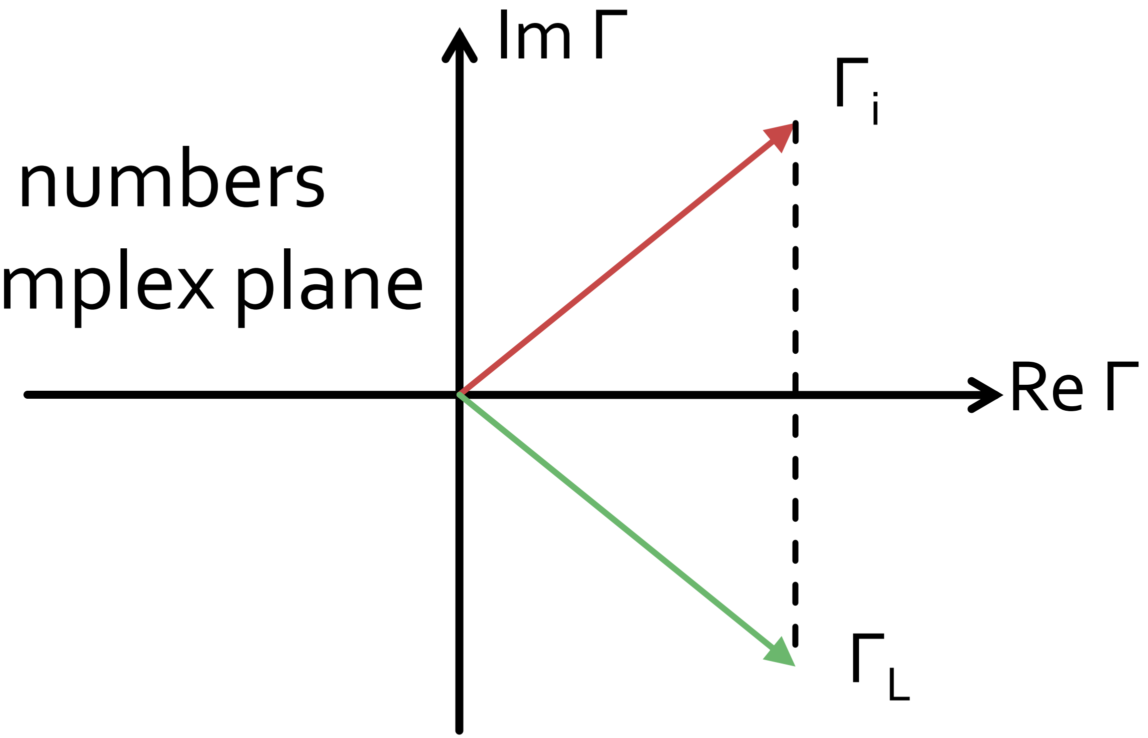
$$Z_L = Z_i^*$$

If we choose a real Zo

$$\Gamma = \frac{Z - Z_0}{Z + Z_0}$$

$$\Gamma_L = \Gamma_i^*$$

- complex numbers
- in the complex plane

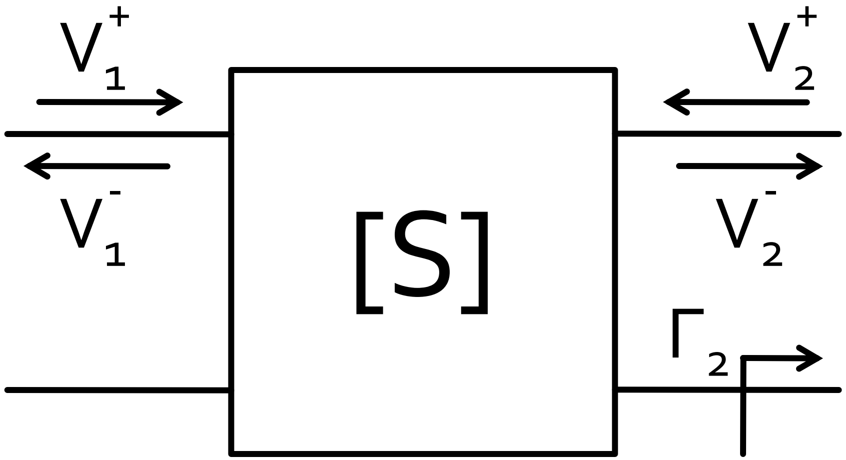


Lecture 3-4

Microwave Network Analysis

Scattering matrix – S

- Scattering parameters



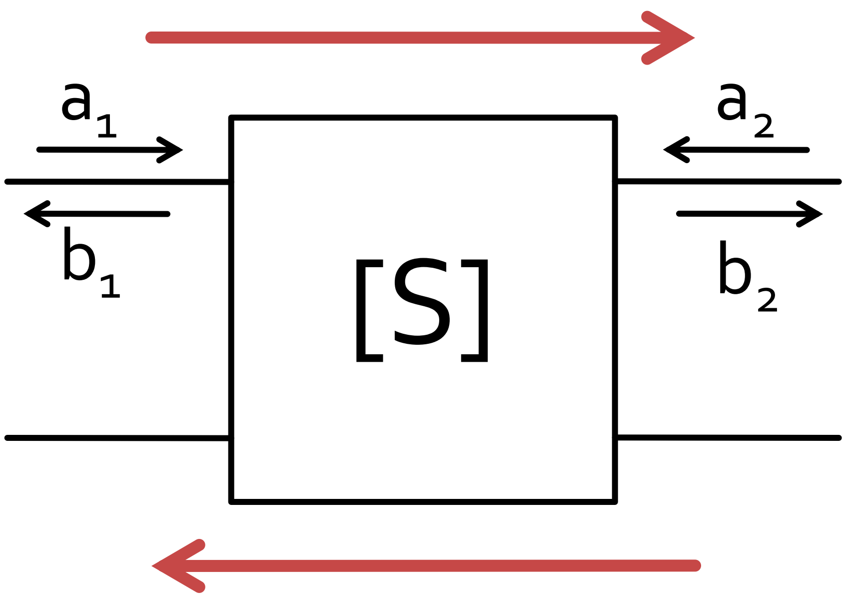
$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0} \qquad S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0}$$

- $V_2^+ = 0$ meaning: port 2 is terminated in matched load to avoid reflections towards the port

$$\Gamma_2 = 0 \rightarrow V_2^+ = 0$$

Scattering matrix – S



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

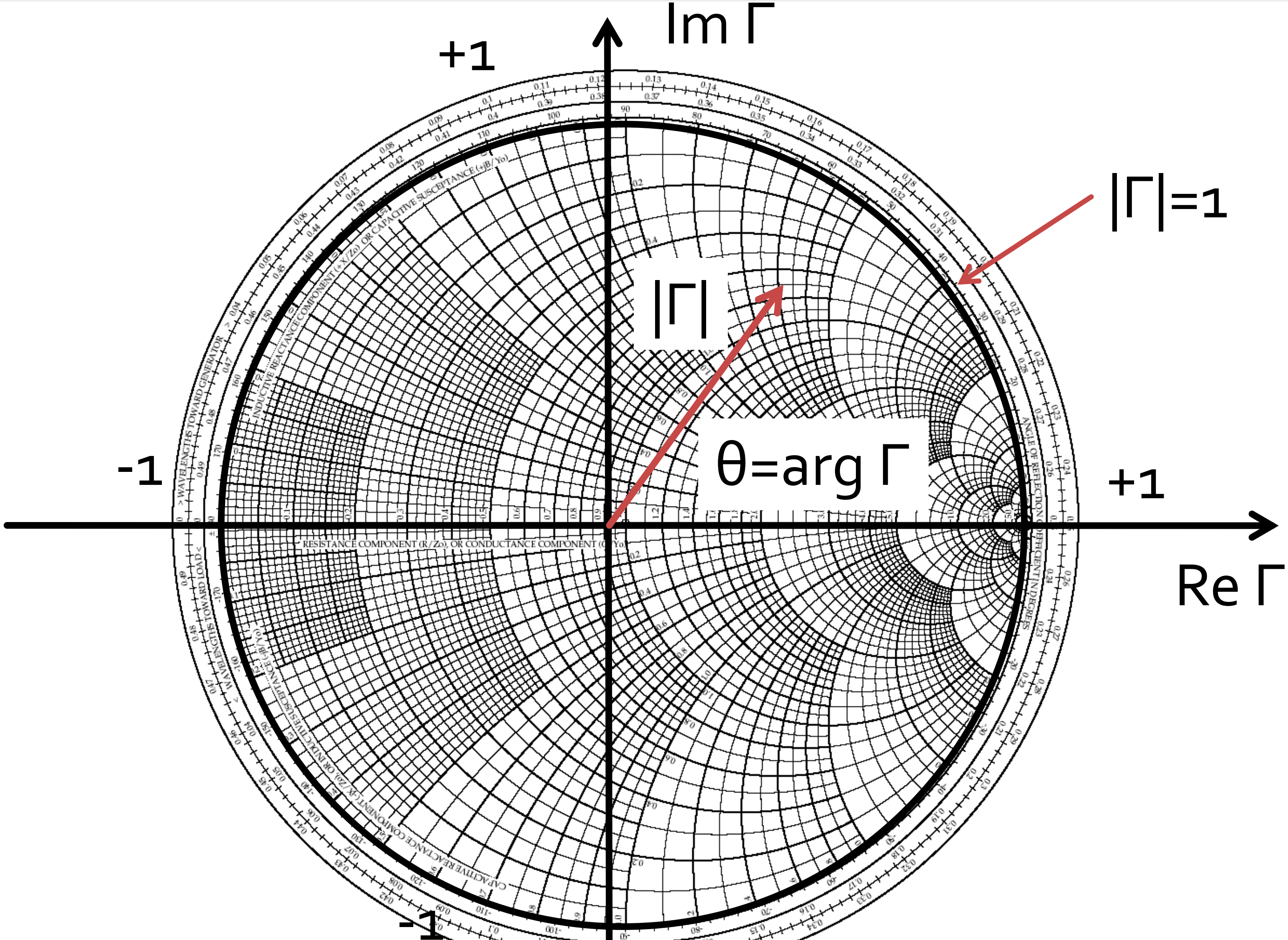
$$|S_{21}|^2 = \frac{Power\ in\ Z_0\ load}{Power\ \ from\ Z_0\ source}$$

- a,b
  - information about signal power **AND** signal phase
- S_{ij}
  - network effect (gain) over signal power **including** phase information

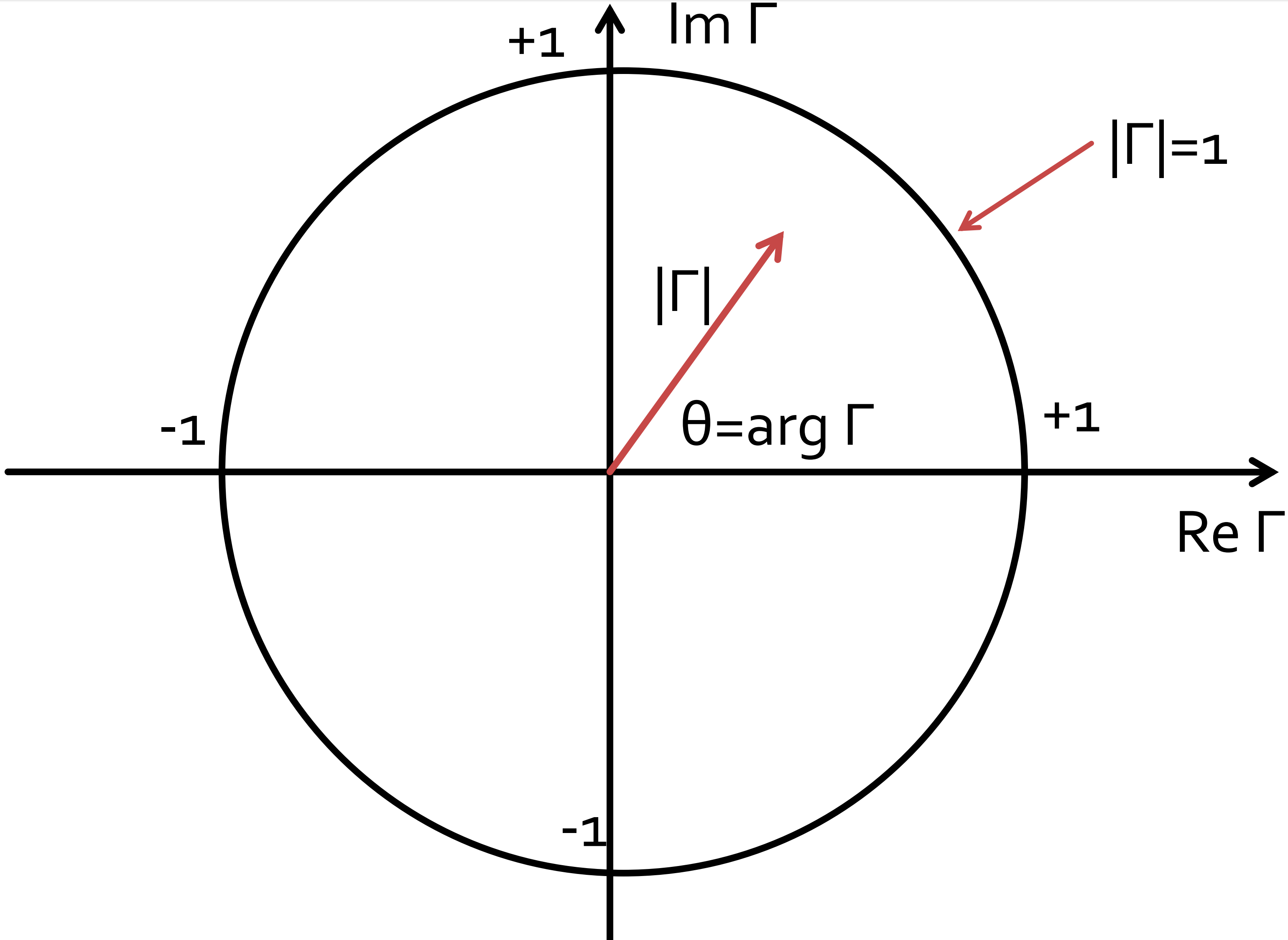
Impedance Matching

The Smith Chart

The Smith Chart



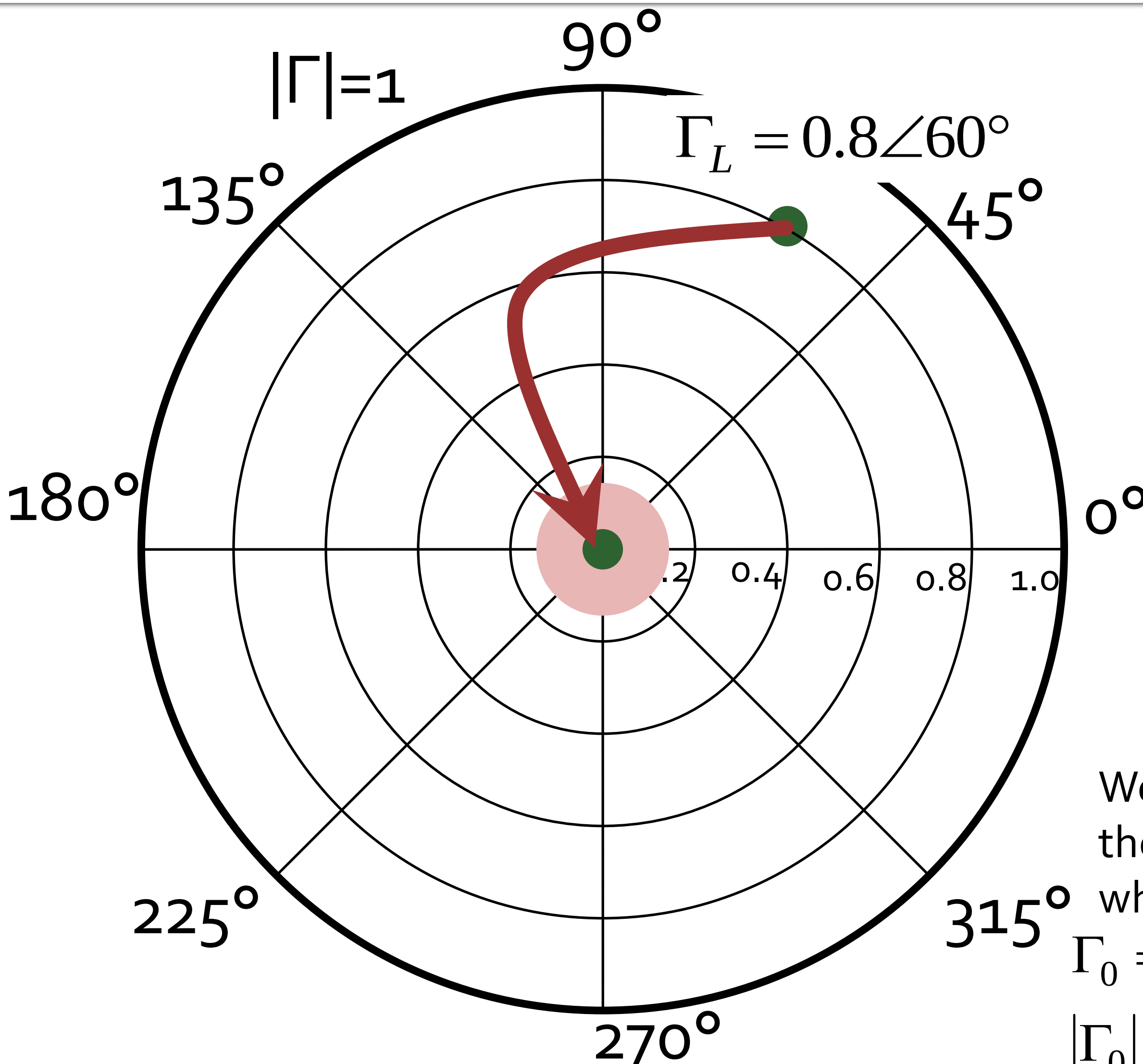
The Smith Chart

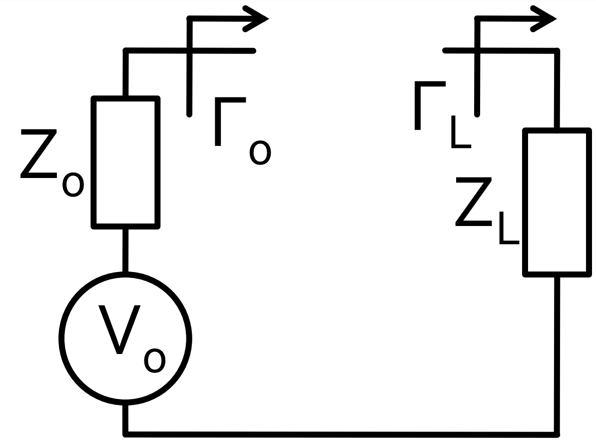


Impedance matching with lumped elements (L Networks)

Impedance Matching

The Smith Chart, reflection coefficient, impedance matching





Matching Z_L load to Z_o source.
We normalize Z_L over Z_o

$$Z_L = 21.429\Omega + j \cdot 82.479\Omega$$

$$z_L = 0.429 + j \cdot 1.65$$

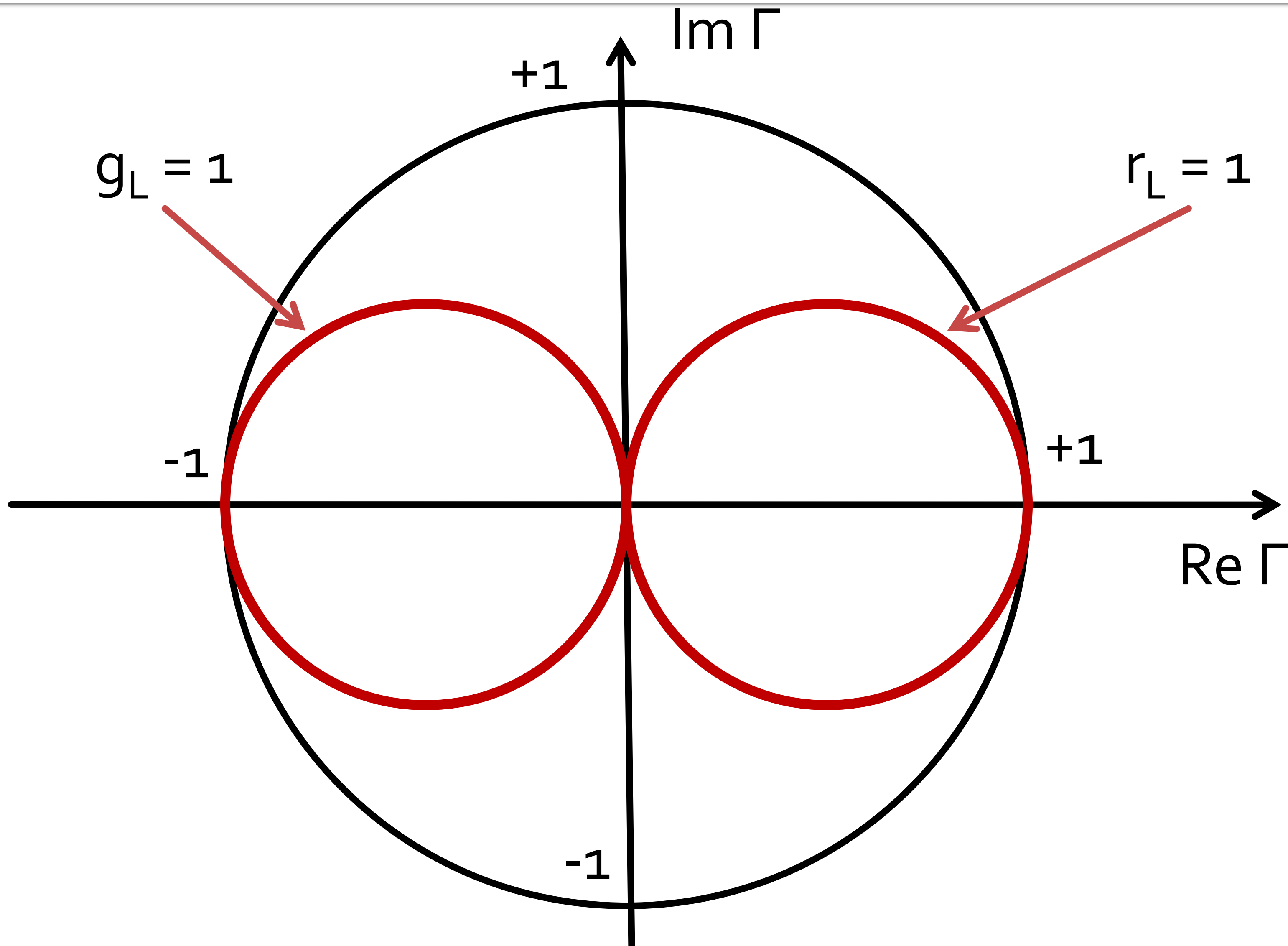
$$\Gamma_L = 0.8\angle 60°$$

We must move the point denoting the reflection coefficient in the area where with a Z_o source we have:

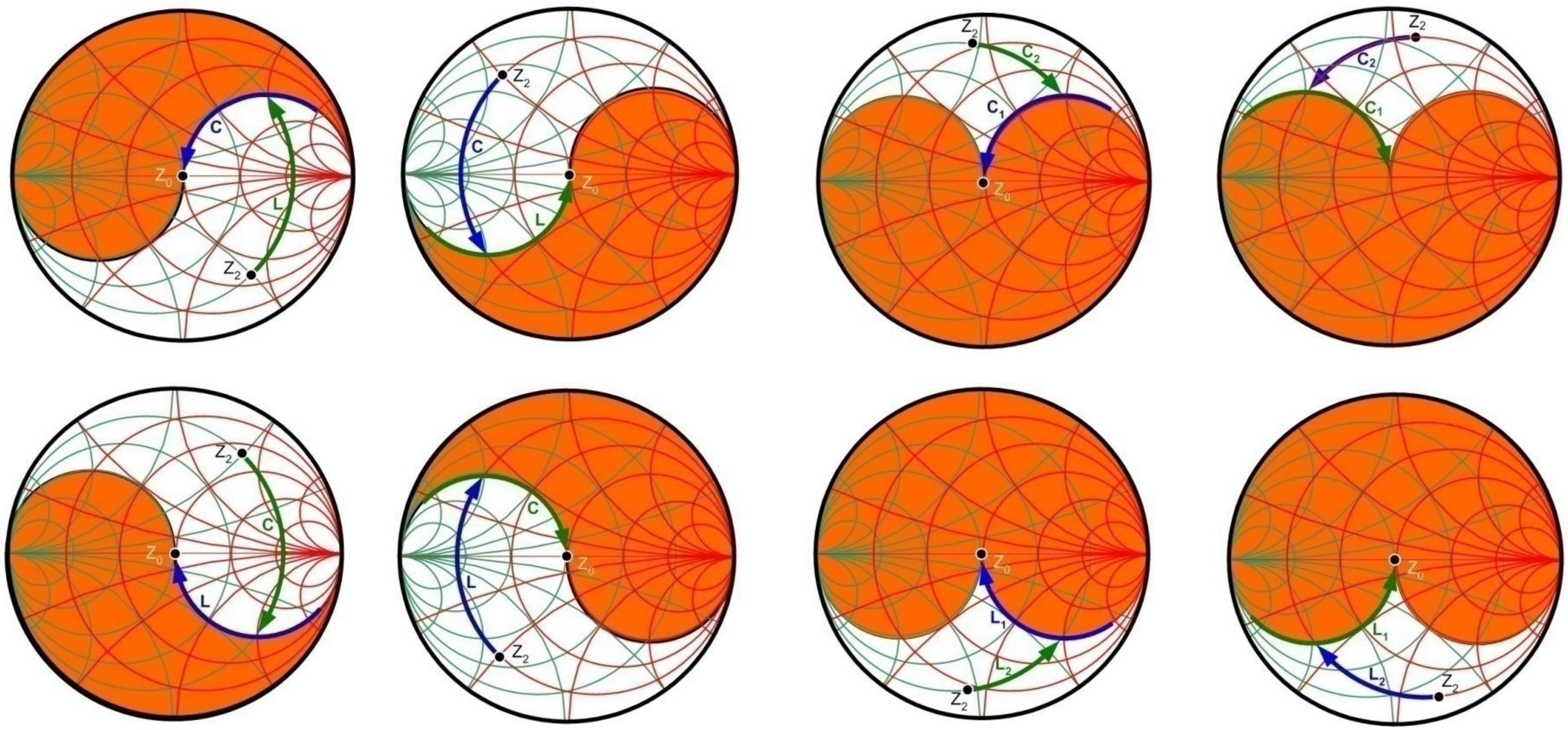
$\Gamma_0 = 0$ perfect match

$|\Gamma_0| \leq \Gamma_m$ "good enough" match

Smith chart, r=1 and g=1



Matching with 2 reactive elements (L Networks)

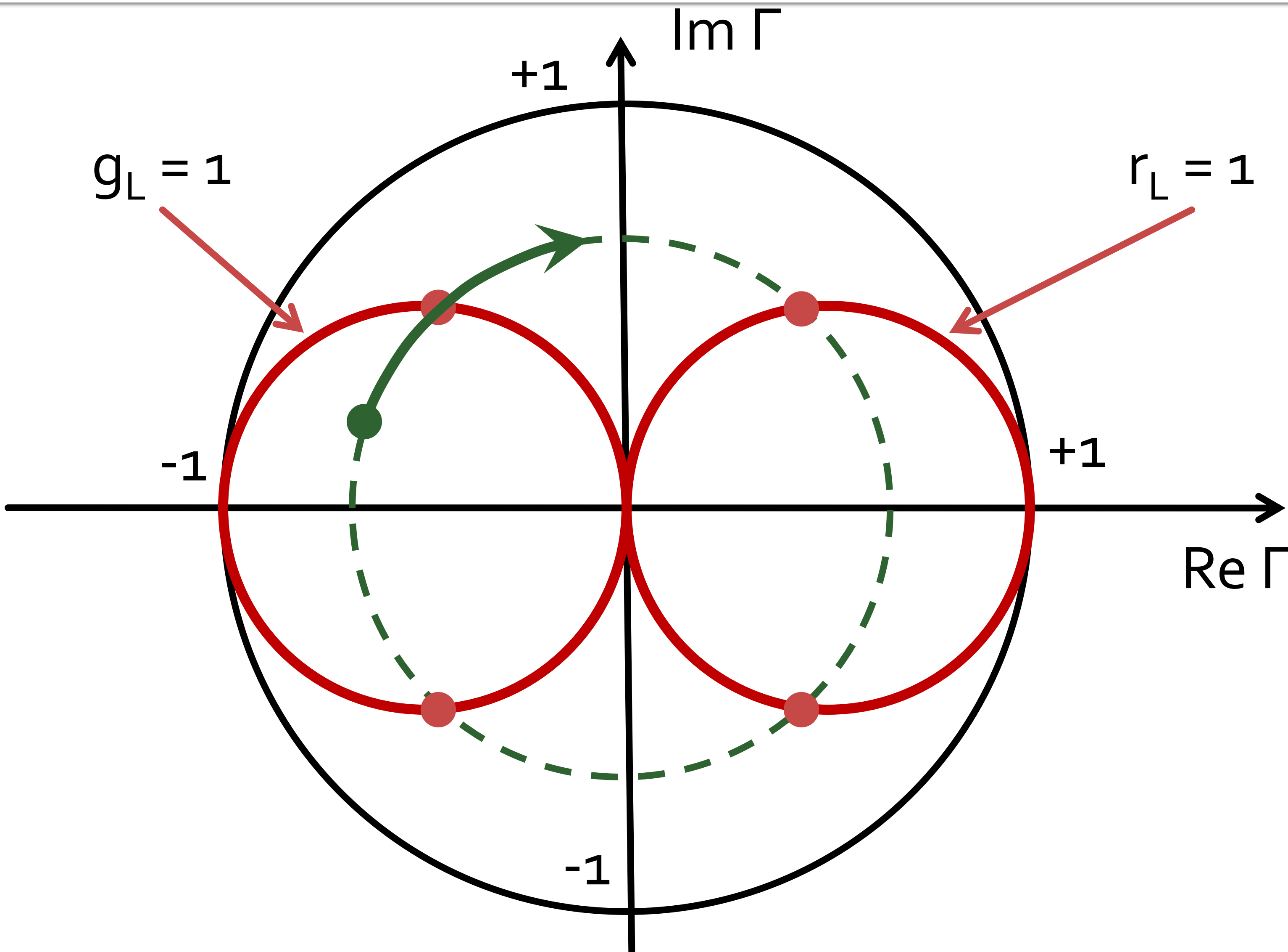


Forbidden area for current network

Impedance Matching with Stubs

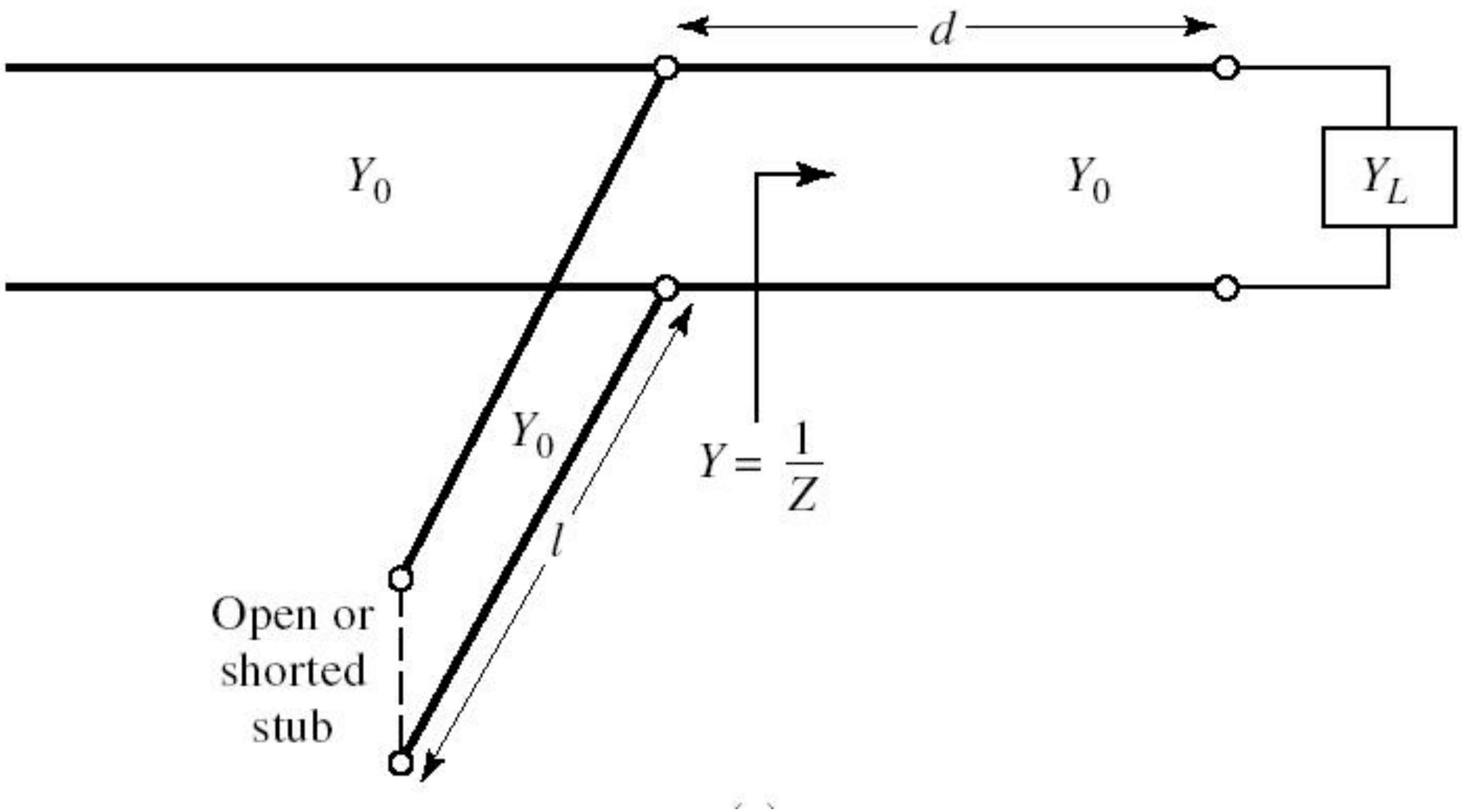
Impedance Matching

Smith chart, r=1 and g=1



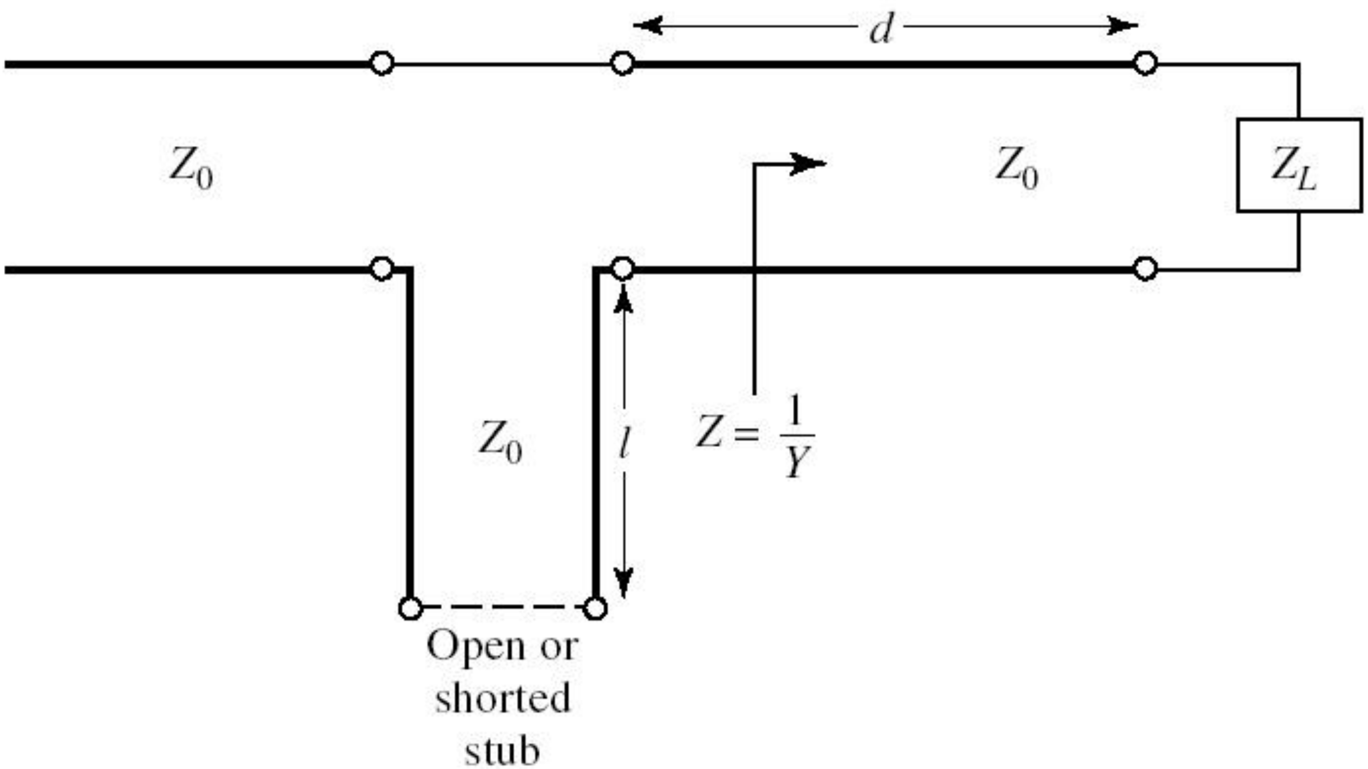
Single stub tuning

- Shunt Stub



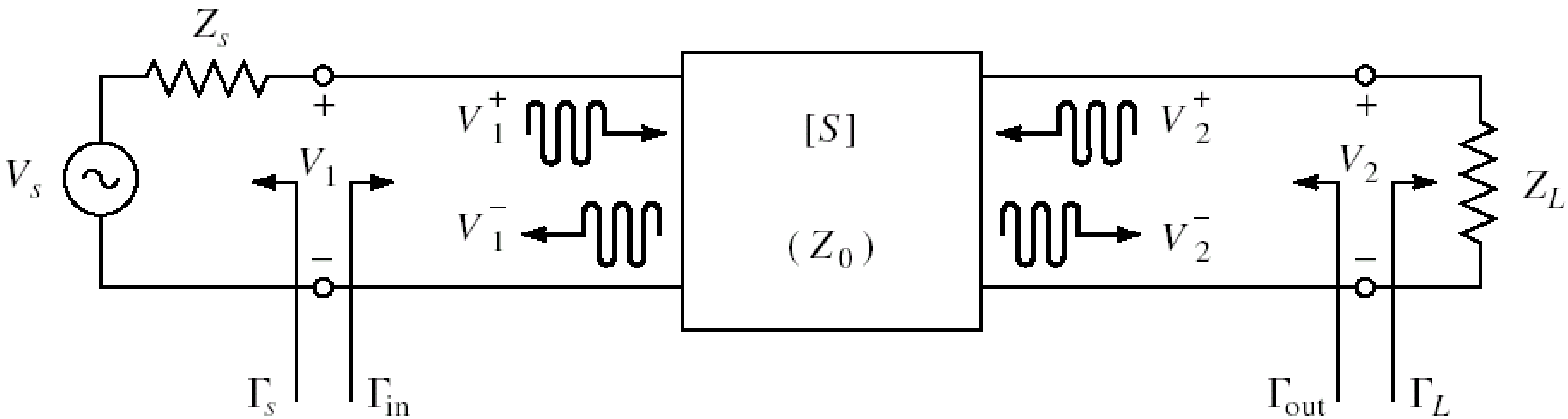
Single stub tuning

- Series Stub
- difficult to realize in single conductor line technologies (microstrip)



Microwave Amplifiers

Amplifier as two-port



- Charaterized with S parameters
- normalized at Zo (implicit 50Ω)
- Datasheets: S parameters for specific bias conditions

Datasheets

NE46100

V_{CE} = 5 V, I_C = 50 mA

FREQUENCY (MHz)	S_{11} MAG	S_{11} ANG	S_{21} MAG	S_{21} ANG	S_{12} MAG	S_{12} ANG	S_{22} MAG	S_{22} ANG	K	MAG^2 (dB)
100	0.778	-137	26.776	114	0.028	30	0.555	-102	0.16	29.8
200	0.815	-159	14.407	100	0.035	29	0.434	-135	0.36	26.2
500	0.826	-177	5.855	84	0.040	38	0.400	-162	0.75	21.7
800	0.827	176	3.682	76	0.052	43	0.402	-169	0.91	18.5
1000	0.826	173	2.963	71	0.058	47	0.405	-172	1.02	16.3
1200	0.825	170	2.441	66	0.064	47	0.412	-174	1.08	14.0
1400	0.820	167	2.111	61	0.069	47	0.413	-176	1.17	12.4
1600	0.828	165	1.863	57	0.078	54	0.426	-177	1.15	11.4
1800	0.827	162	1.671	53	0.087	50	0.432	-178	1.14	10.6
2000	0.828	159	1.484	49	0.093	50	0.431	-180	1.17	9.5
2500	0.822	153	1.218	39	0.11	48	0.462	177	1.18	7.8
3000	0.818	148	1.010	30	0.135	46	0.490	174	1.16	6.3
3500	0.824	142	0.876	21	0.147	44	0.507	170	1.16	5.3
4000	0.812	137	0.762	13	0.168	38	0.535	167	1.14	4.3

V_{CE} = 5 V, I_C = 100 mA

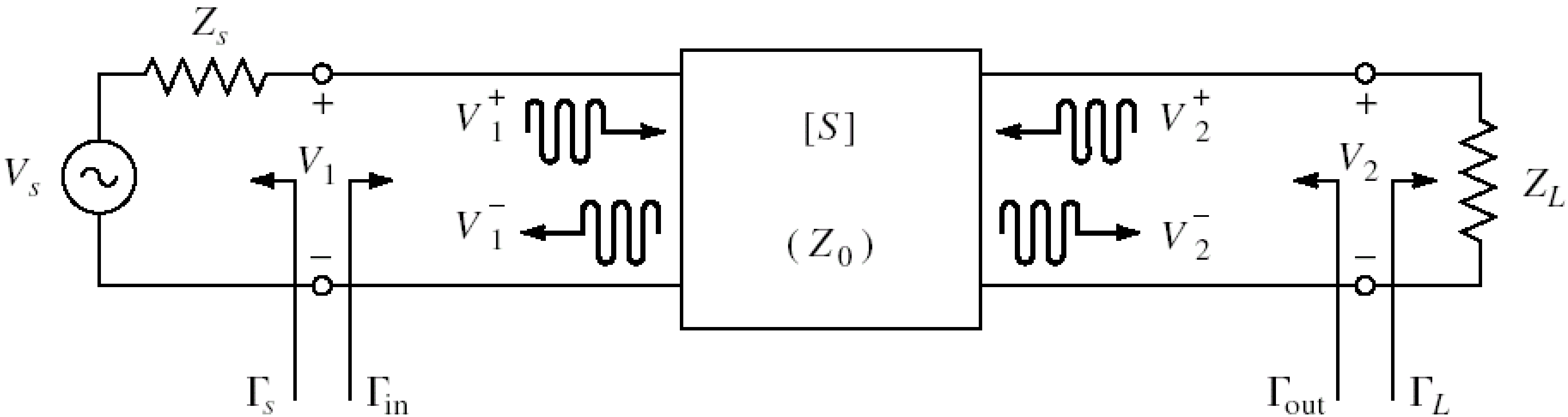
FREQUENCY (MHz)	S_{11} MAG	S_{11} ANG	S_{21} MAG	S_{21} ANG	S_{12} MAG	S_{12} ANG	S_{22} MAG	S_{22} ANG	K	MAG^2 (dB)
100	0.778	-144	27.669	111	0.027	35	0.523	-114	0.27	30.2
200	0.820	-164	14.559	97	0.029	29	0.445	-144	0.42	27.0
500	0.832	-179	5.885	84	0.035	38	0.435	-166	0.81	22.2
800	0.833	175	3.691	76	0.048	45	0.435	-173	0.95	18.8
1000	0.831	172	2.980	71	0.056	51	0.437	-176	1.05	16.0
1200	0.836	169	2.464	67	0.061	52	0.432	-178	1.11	14.0
1400	0.829	166	2.121	61	0.072	53	0.447	-180	1.12	12.6
1600	0.831	164	1.867	58	0.080	54	0.445	179	1.14	11.4

S2P - Touchstone

- Touchstone file format (*.s2p)

```
! SIEMENS Small Signal Semiconductors
! VDS = 3.5 V     ID = 15 mA
# GHz  S  MA  R  50
! f        S11          S21          S12          S22
! GHz    MAG   ANG     MAG   ANG     MAG   ANG     MAG   ANG
 1.000 0.9800  -18.0   2.230  157.0  0.0240   74.0  0.6900  -15.0
 2.000 0.9500  -39.0   2.220  136.0  0.0450   57.0  0.6600  -30.0
 3.000 0.8900  -64.0   2.210  110.0  0.0680   40.0  0.6100  -45.0
 4.000 0.8200  -89.0   2.230   86.0  0.0850   23.0  0.5600  -62.0
 5.000 0.7400 -115.0   2.190   61.0  0.0990    7.0  0.4900  -80.0
 6.000 0.6500 -142.0   2.110   36.0  0.1070  -10.0  0.4100  -98.0
!
! f      Fmin   Gammaopt  rn/50
! GHz      dB    MAG  ANG    -
 2.000     1.00  0.72   27   0.84
 4.000     1.40  0.64   61   0.58
```

Amplifier as two-port



$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$

$$\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}$$

Two-Port Power Gains

- Power Gain

$$G=\frac{P_L}{P_{in}}=\frac{|S_{21}|^2\cdot\left(1-|\Gamma_L|^2\right)}{\left(1-|\Gamma_{in}|^2\right)\cdot|1-S_{22}\cdot\Gamma_L|^2}$$

$$P_{in}=P_{in}\left(\Gamma_S,\Gamma_{in}(\Gamma_L),S\right)$$

$$P_L=P_L\left(\Gamma_S,\Gamma_{in}(\Gamma_L),S\right)$$

- The **actual** power gain **introduced** by the amplifier is less important because a higher gain may be accompanied by a **decrease** in input power (power actually drained from the source)
- We prefer to characterize the amplifier effect looking to the **power actually delivered to the load** in relation to the power **available from the source** (which is a constant)

Two-Port Power Gains

- **Available** power gain

$$G_A = \frac{P_{av\,L}}{P_{av\,S}} = \frac{|S_{21}|^2 \cdot \left(1-|\Gamma_S|^2\right)}{|1-S_{22}\cdot\Gamma_L|^2 \cdot \left(1-|\Gamma_{out}|^2\right)}$$

- **Transducer** power gain

$$G_T = \frac{P_L}{P_{av\,S}} = \frac{|S_{21}|^2 \cdot \left(1-|\Gamma_S|^2\right)\cdot\left(1-|\Gamma_L|^2\right)}{|1-\Gamma_S\cdot\Gamma_{in}|^2 \cdot |1-S_{22}\cdot\Gamma_L|^2}$$

$$\Gamma_{in} = \Gamma_{in}(\Gamma_L)$$

- **Unilateral transducer** power gain

$$G_{TU} = |S_{21}|^2 \cdot \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2} \cdot \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

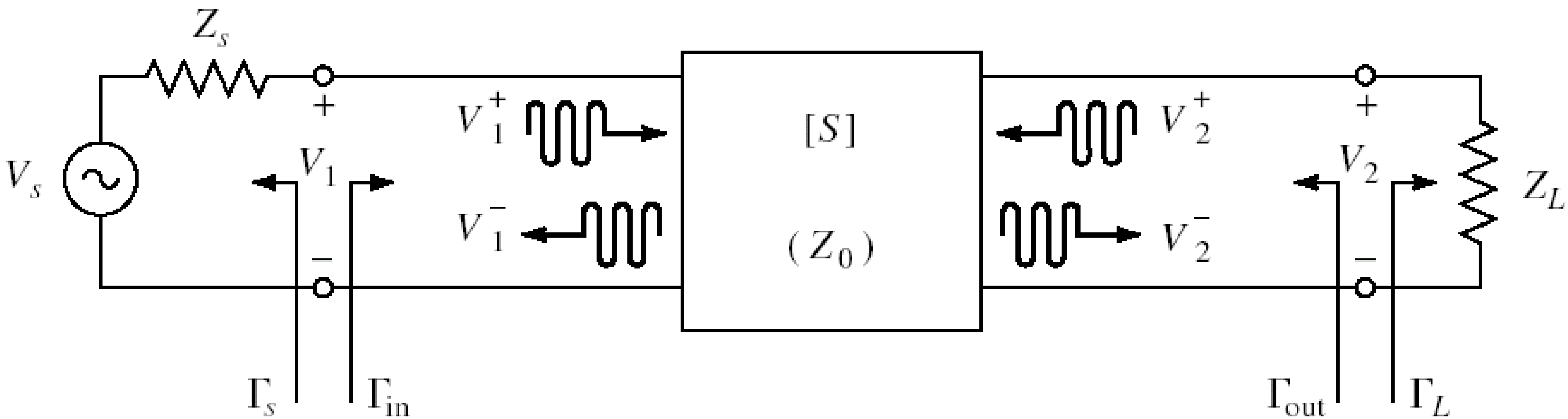
$$S_{12} \cong 0 \qquad \Gamma_{in} = S_{11}$$

Input and output can be treated independently

Stability

Microwave Amplifiers

Amplifier as two-port



- For an amplifier two-port we are interested in:
  - **stability**
  - power gain
  - noise (sometimes – small signals)
  - linearity (sometimes – large signals)

Stability

$$|\Gamma_{in}| < 1 \qquad \left| S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L} \right| < 1$$

- We can calculate conditions to be met by Γ_L to achieve stability

$$|\Gamma_{out}| < 1 \qquad \left| S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S} \right| < 1$$

- We can calculate conditions to be met by Γ_S to achieve stability

Stability

$$|\Gamma_{in}| < 1 \qquad \left| S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L} \right| < 1$$

- The limit between stability/instability

$$|\Gamma_{in}| = 1 \qquad \log_{10}|\Gamma_{in}| = 0 \qquad \left| S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L} \right| = 1$$

$$|S_{11} \cdot (1 - S_{22} \cdot \Gamma_L) + S_{12} \cdot S_{21} \cdot \Gamma_L| = |1 - S_{22} \cdot \Gamma_L|$$

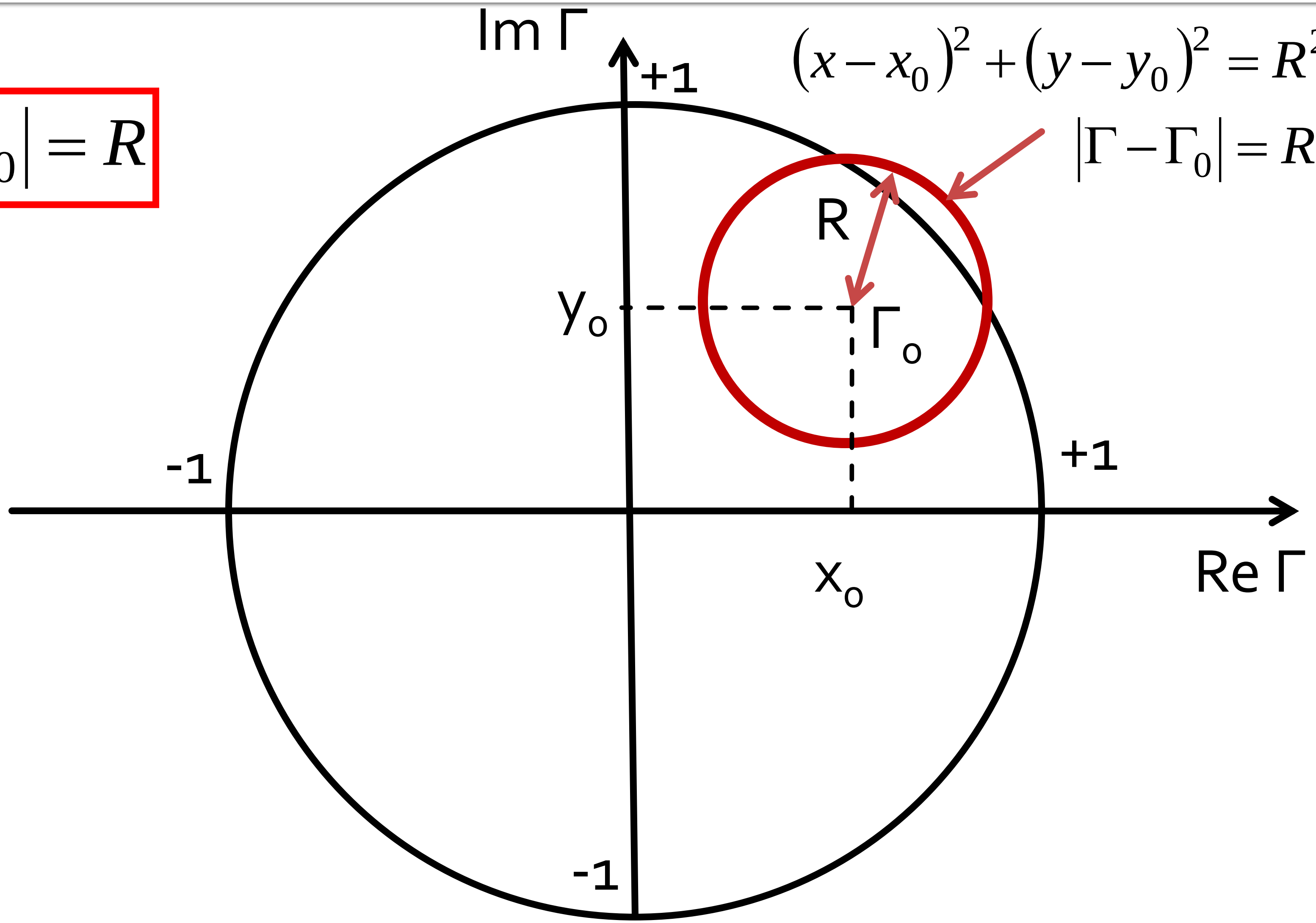
- determinant of the S matrix $\Delta = S_{11} \cdot S_{22} - S_{12} \cdot S_{21}$

$$|S_{11} - \Delta \cdot \Gamma_L| = |1 - S_{22} \cdot \Gamma_L|$$

$$|S_{11} - \Delta \cdot \Gamma_L|^2 = |1 - S_{22} \cdot \Gamma_L|^2$$

Stability

$$\left|\Gamma - \Gamma_0\right| = R$$



Output stability circle (CSOUT)

$$\left|\Gamma_L - \frac{\left(S_{22} - \Delta \cdot S_{11}^*\right)^*}{|S_{22}|^2 - |\Delta|^2}\right| = \left|\frac{S_{12} \cdot S_{21}}{|S_{22}|^2 - |\Delta|^2}\right| \qquad |\Gamma_L - C_L| = R_L$$

- We obtain the equation of a circle in the complex plane, which represents the locus of Γ_L for the **limit between stability and instability** ($|\Gamma_{in}| = 1$)
- This circle is the **output stability circle** (Γ_L)

$$C_L = \frac{\left(S_{22} - \Delta \cdot S_{11}^*\right)^*}{|S_{22}|^2 - |\Delta|^2} \qquad R_L = \frac{|S_{12} \cdot S_{21}|}{\left||S_{22}|^2 - |\Delta|^2\right|}$$

Input stability circle (CSIN)

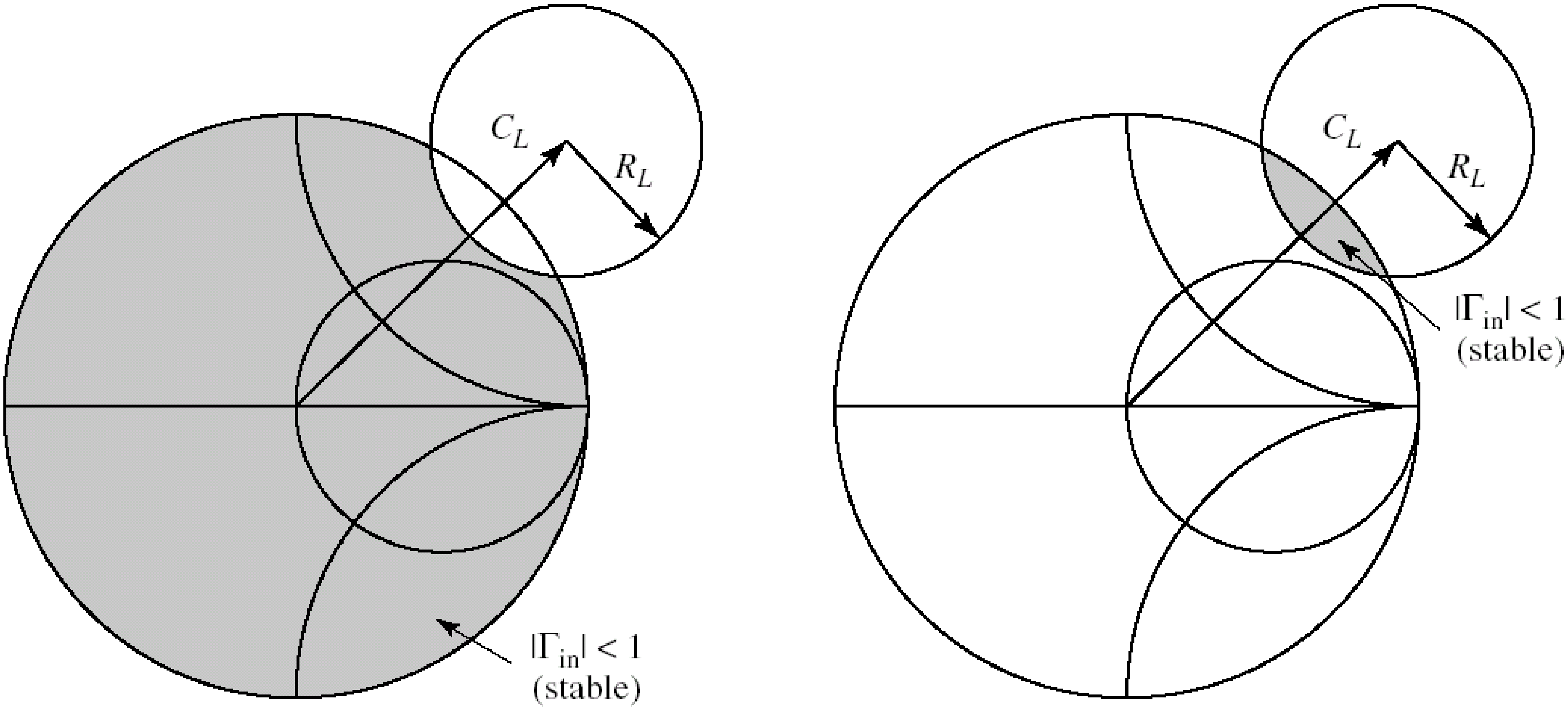
- Similarly

$$|\Gamma_{out}| = 1 \qquad \left| S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S} \right| = 1$$

- We obtain the equation of a circle in the complex plane, which represents the locus of Γ_S for the **limit between stability and instability** ($|\Gamma_{out}| = 1$)
- This circle is the **input stability circle** (Γ_S)

$$C_S = \frac{\left(S_{11} - \Delta \cdot S_{22}^*\right)^*}{|S_{11}|^2 - |\Delta|^2} \qquad R_S = \frac{|S_{12} \cdot S_{21}|}{\left||S_{11}|^2 - |\Delta|^2\right|}$$

Output stability circle (CSOUT)



(a) (b)

- Two cases possible: (a) stable outside/ (b) stable inside

Output stability circle (CSOUT)

- Identification of the stability / instability regions
  - Center of the Smith Chart in Γ_L complex plane correspond to $\Gamma_L = 0$
  - Input reflection coefficient

$$\Gamma_{in} = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L} \qquad \Gamma_{in}\big|_{\Gamma_L = 0} = S_{11} \qquad \left|\Gamma_{in}\right|_{\Gamma_L = 0} = \left|S_{11}\right|$$

  - A decision can be made based on |S11| value the position of the center of the Smith chart (origin of the complex plane) relative to the circle

Identification of the stability / instability regions

- Output stability circle
  - **|S11| < 1** → the center of the Smith chart on which Γ_L is represented is a **stable point**, so it's placed in the stability region (most often situation)
  - |S11| > 1 → the center of the Smith chart on which Γ_L is represented is a **unstable point**, so it's placed in the instability region
- Input stability circle
  - **|S22| < 1** → the center of the Smith chart on which Γ_S is represented is a **stable point**, so it's placed in the stability region (most often situation)
  - |S22| > 1 → the center of the Smith chart on which Γ_S is represented is a **unstable point**, so it's placed in the instability region

Example

- ATF-34143 **at Vds=3V Id=20mA.**
- @5GHz
  - S11 = 0.64∠139°
  - S12 = 0.119∠-21°
  - S21 = 3.165 ∠16°
  - S22 = 0.22 ∠146°

```
!ATF-34143
!S-PARAMETERS at Vds=3V  Id=20mA.   LAST UPDATED 01-29-99

# ghz s ma r 50

2.0  0.75  -126  6.306  90  0.088  23  0.26  -120
2.5  0.72  -145  5.438  75  0.095  15  0.25  -140
3.0  0.69  -162  4.762  62  0.102  7  0.23  -156
4.0  0.65  166  3.806  38  0.111  -8  0.22  174
5.0  0.64  139  3.165  16  0.119  -21  0.22  146
6.0  0.65  114  2.706  -5  0.125  -35  0.23  118
7.0  0.66  89  2.326  -27  0.129  -49  0.25  91
8.0  0.69  67  2.017  -47  0.133  -62  0.29  67
9.0  0.72  48  1.758  -66  0.135  -75  0.34  46

!FREQ   Fopt   GAMMA OPT      RN/Zo
!GHZ    dB     MAG     ANG     -

2.0  0.19  0.71  66  0.09
2.5  0.23  0.65  83  0.07
3.0  0.29  0.59  102  0.06
4.0  0.42  0.51  138  0.03
5.0  0.54  0.45  174  0.03
6.0  0.67  0.42  -151  0.05
7.0  0.79  0.42  -118  0.10
8.0  0.92  0.45  -88  0.18
9.0  1.04  0.51  -63  0.30
10.0  1.16  0.61  -43  0.46
```

Solution + region identification

- S parameters
  - S11 = -0.483+0.42·j
  - S12 = 0.111-0.043·j
  - S21 = 3.042+0.872·j
  - S22 = -0.182+0.123·j
- | S22 | < 1
- $|C_L| < R_L$, 0∈CSOUT
- The center of the Smith chart is placed inside the output stability circle (0∈CSOUT) and is a stable point
  - the inside of the output stability circle – stability region
  - the outside of the output stability circle – instability region

$$C_L = \frac{\left(S_{22} - \Delta \cdot S_{11}^*\right)^*}{|S_{22}|^2 - |\Delta|^2} = 3.931 - 0.897 \cdot j$$

$$|C_L| = 4.032$$

$$R_L = \frac{|S_{12} \cdot S_{21}|}{\left||S_{22}|^2 - |\Delta|^2\right|} = 4.891$$

Solution + region identification

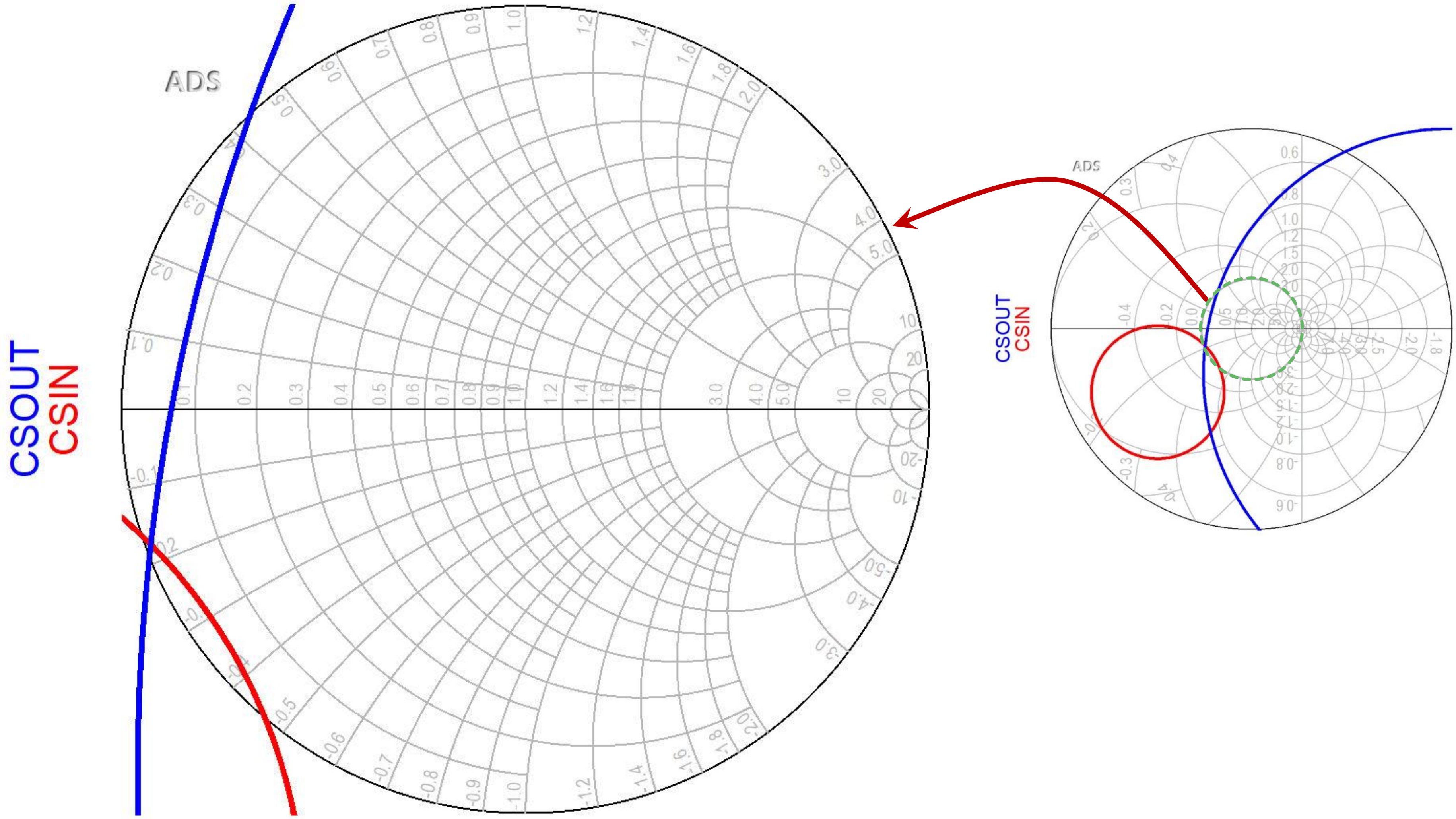
- S parameters
  - S11 = -0.483+0.42·j
  - S12 = 0.111-0.043·j
  - S21 = 3.042+0.872·j
  - S22 = -0.182+0.123·j
- | S11 | < 1
- $|C_S| > R_S$, $0 \notin CSIN$
- The center of the Smith chart is placed outside the input stability circle ($0 \notin CSIN$) and is a stable point
  - the outside of the input stability circle – stability region
  - the inside of the input stability circle – instability region

$$C_S = \frac{\left(S_{11} - \Delta \cdot S_{22}^*\right)^*}{|S_{11}|^2 - |\Delta|^2} = -1.871 - 1.265 \cdot j$$

$$|C_S| = 2.259$$

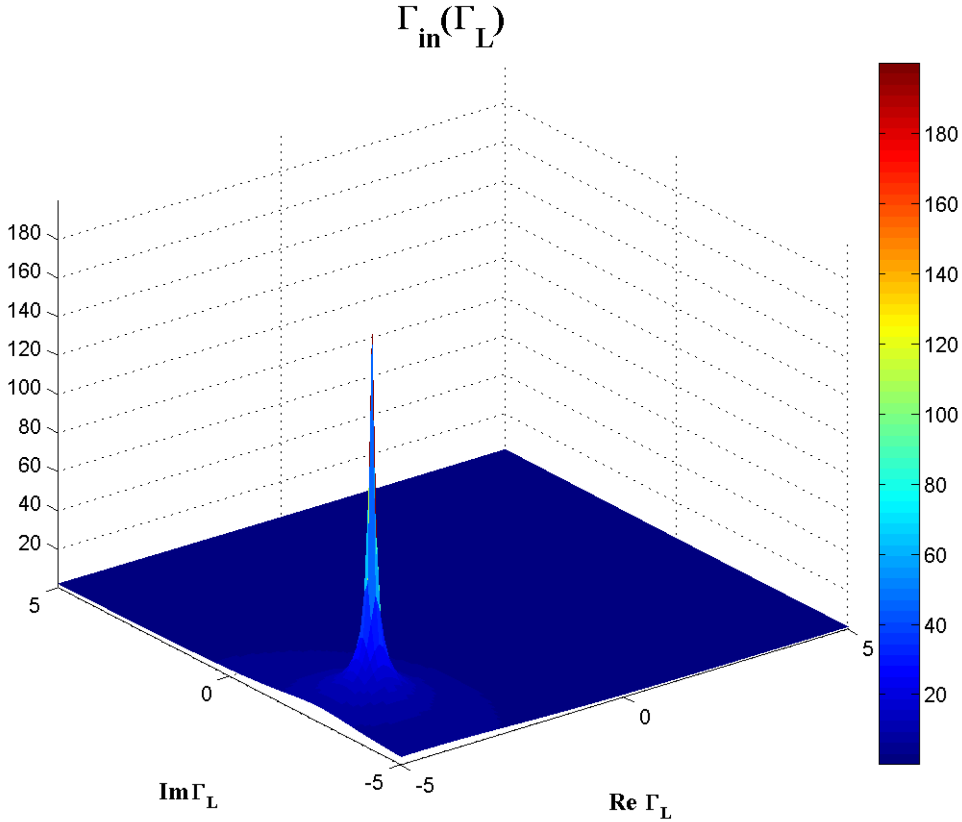
$$R_S = \frac{|S_{12} \cdot S_{21}|}{\left||S_{11}|^2 - |\Delta|^2\right|} = 1.325$$

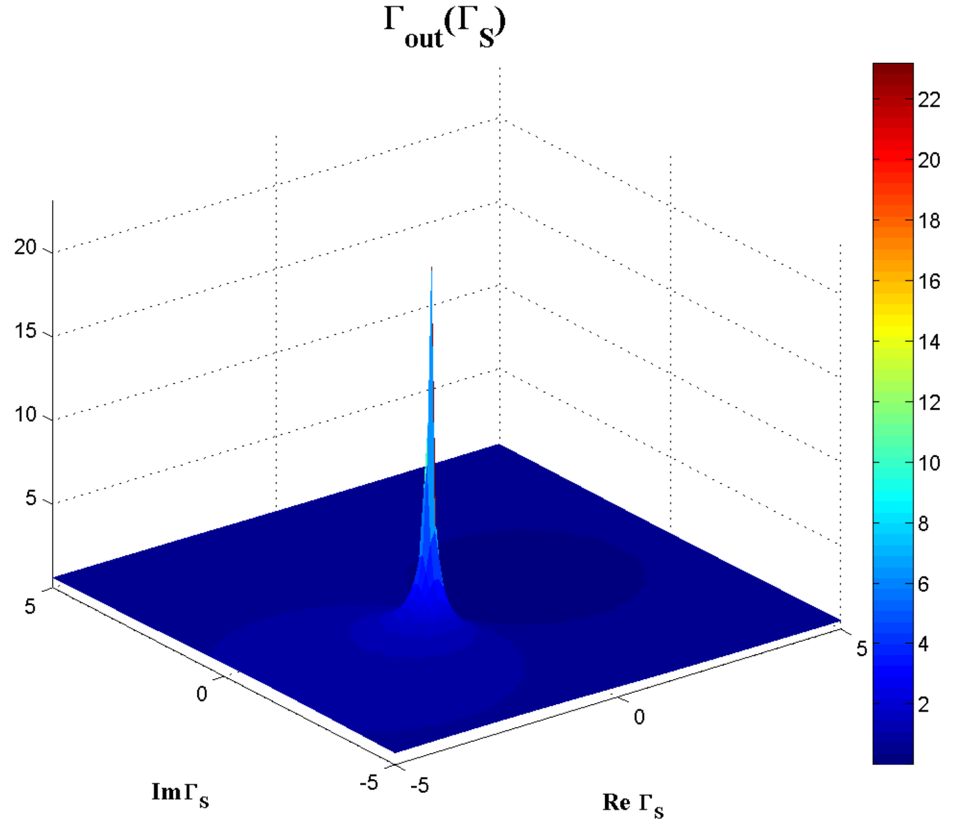
ADS



3D representation of $|\Gamma_{in}|$, $|\Gamma_{out}|$

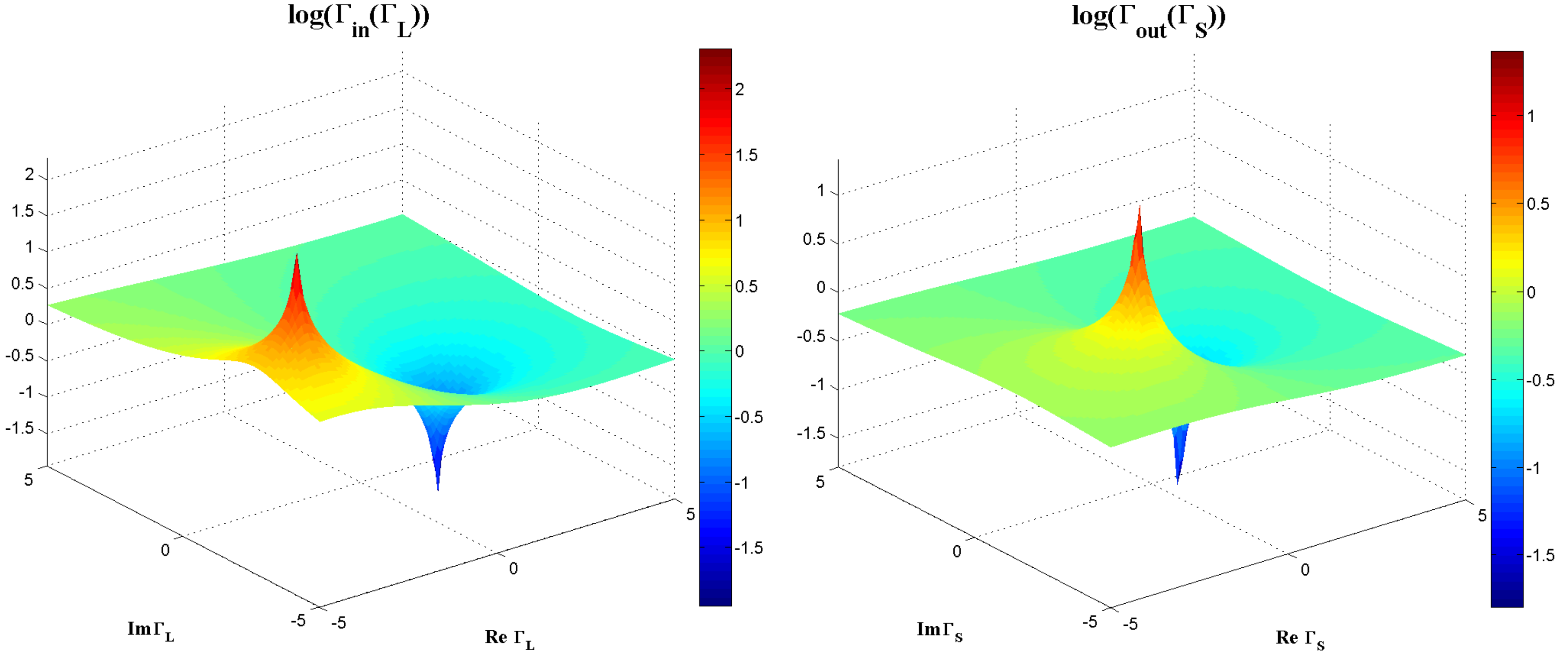
- High variations -> we change to z logarithmic scale





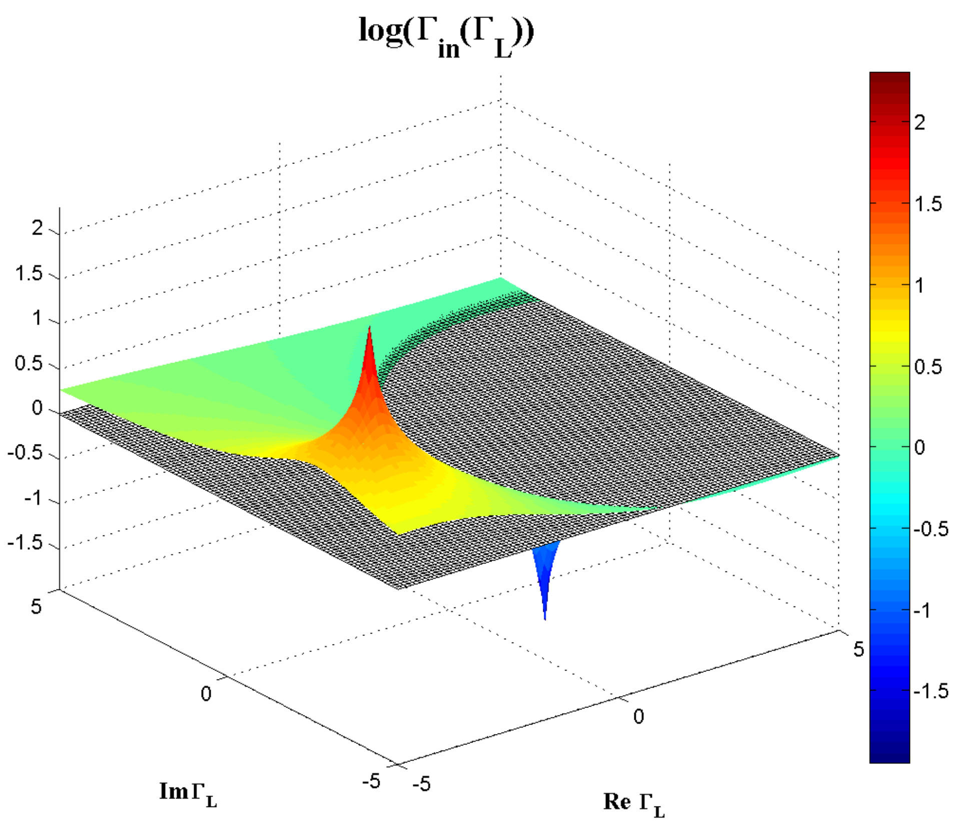
3D representation of $|\Gamma_{in}|$, $|\Gamma_{out}|$

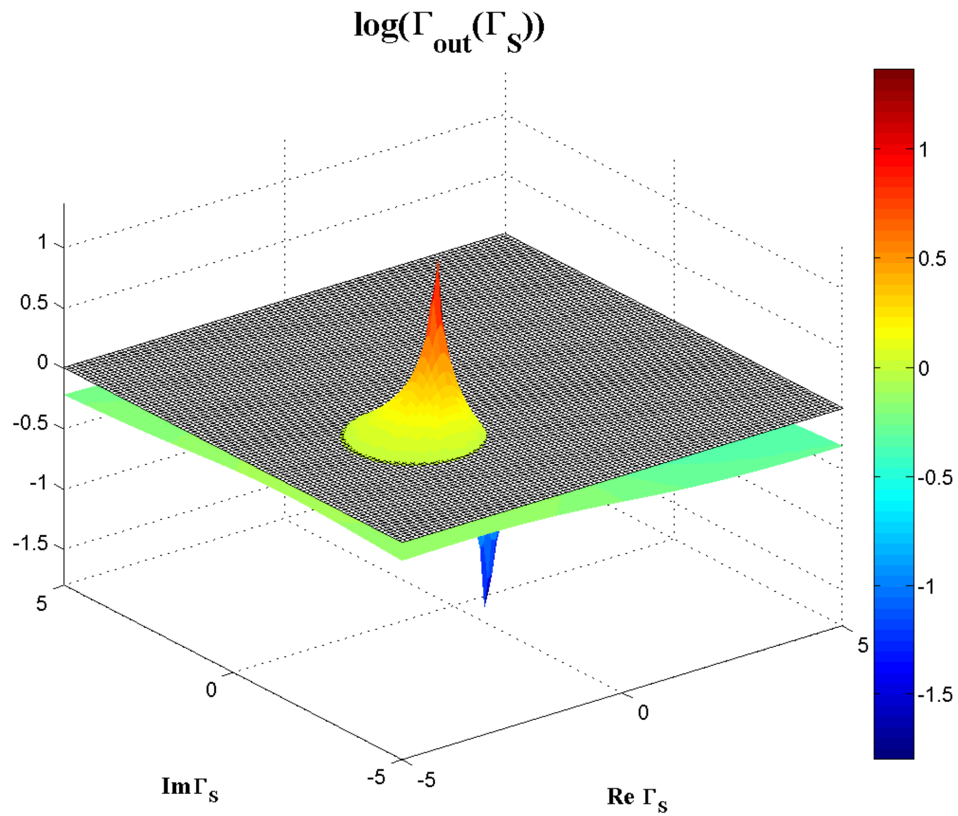
- $\log_{10}|\Gamma_{in}|$, $\log_{10}|\Gamma_{out}|$



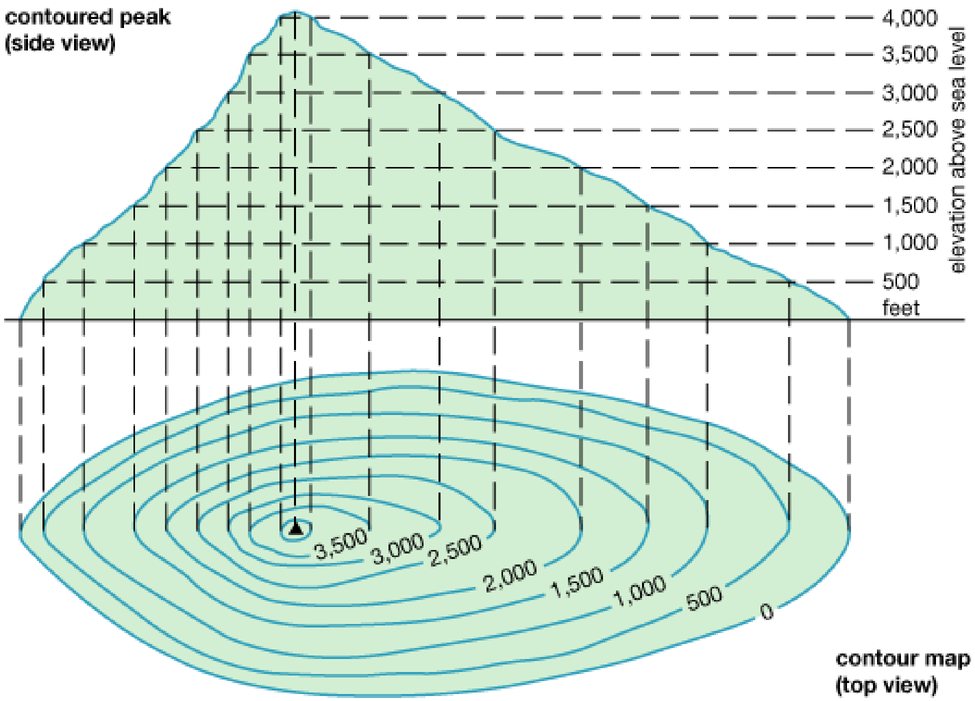
3D representation of $|\Gamma_{in}|$, $|\Gamma_{out}|$, $|\Gamma|=1$

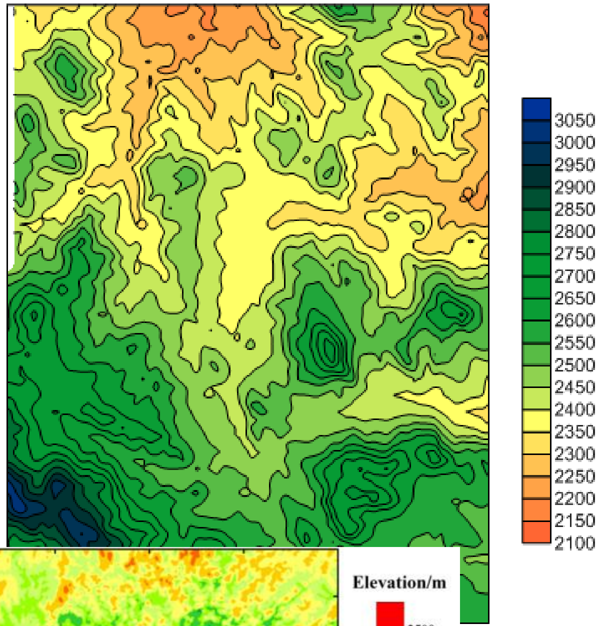
- $|\Gamma| = 1 \rightarrow \log_{10}|\Gamma| = 0$, the intersection with the plane z = 0 is a circle

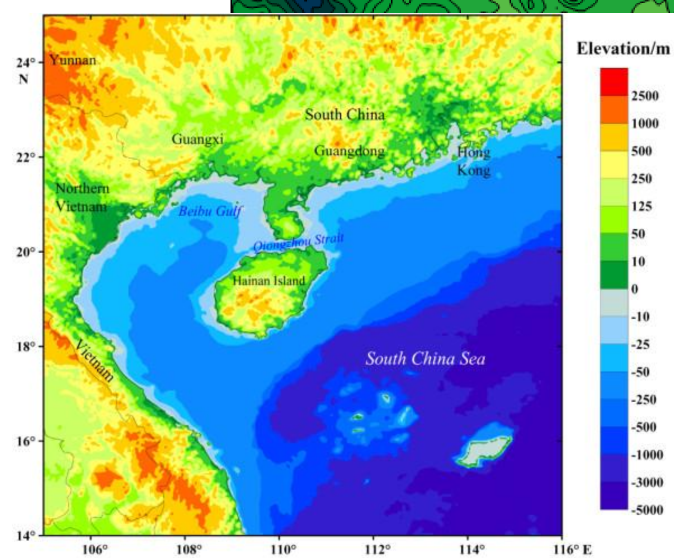




Contour map/lines

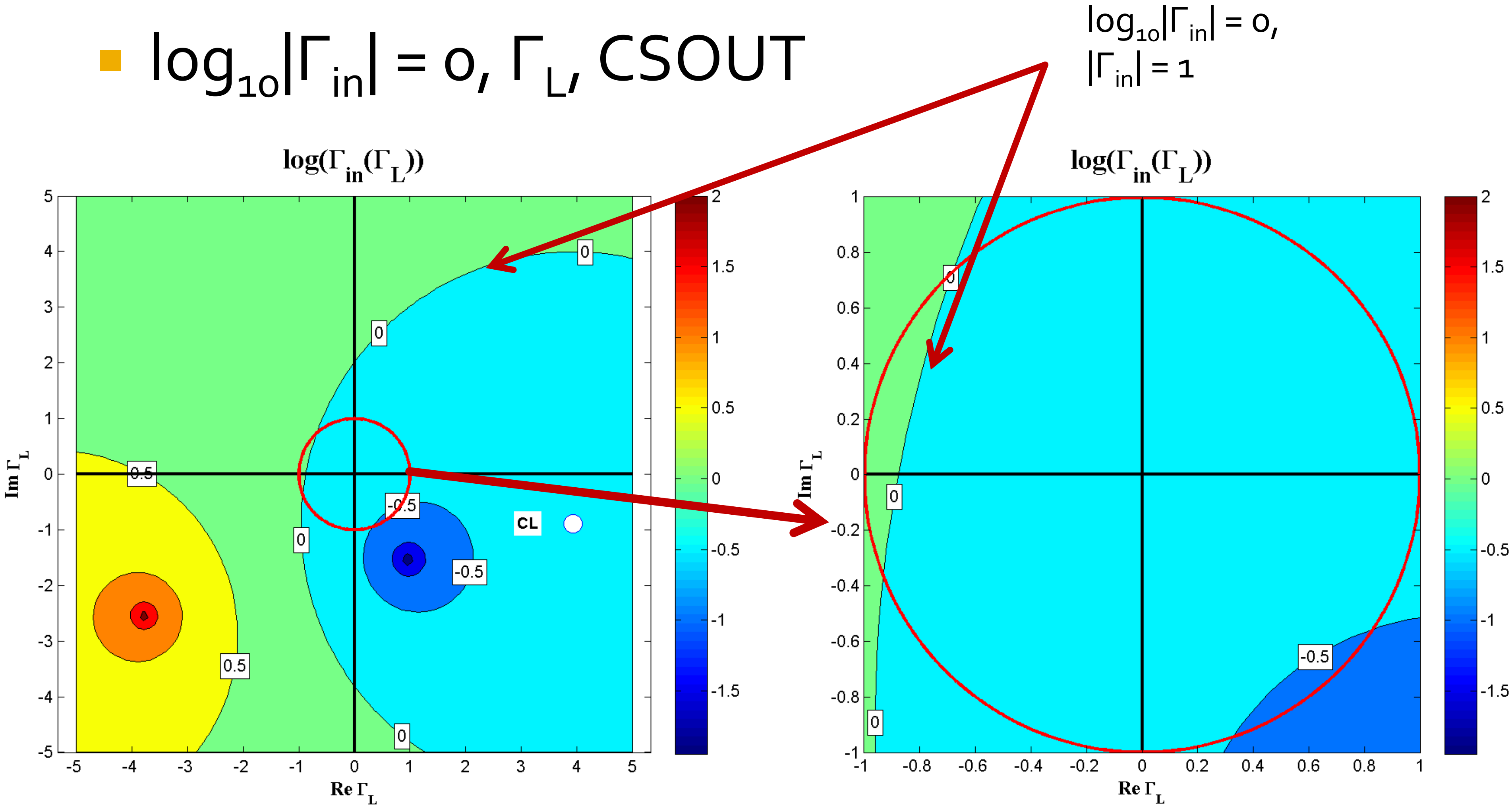






Contour lines of $\log_{10}|\Gamma_{in}|$

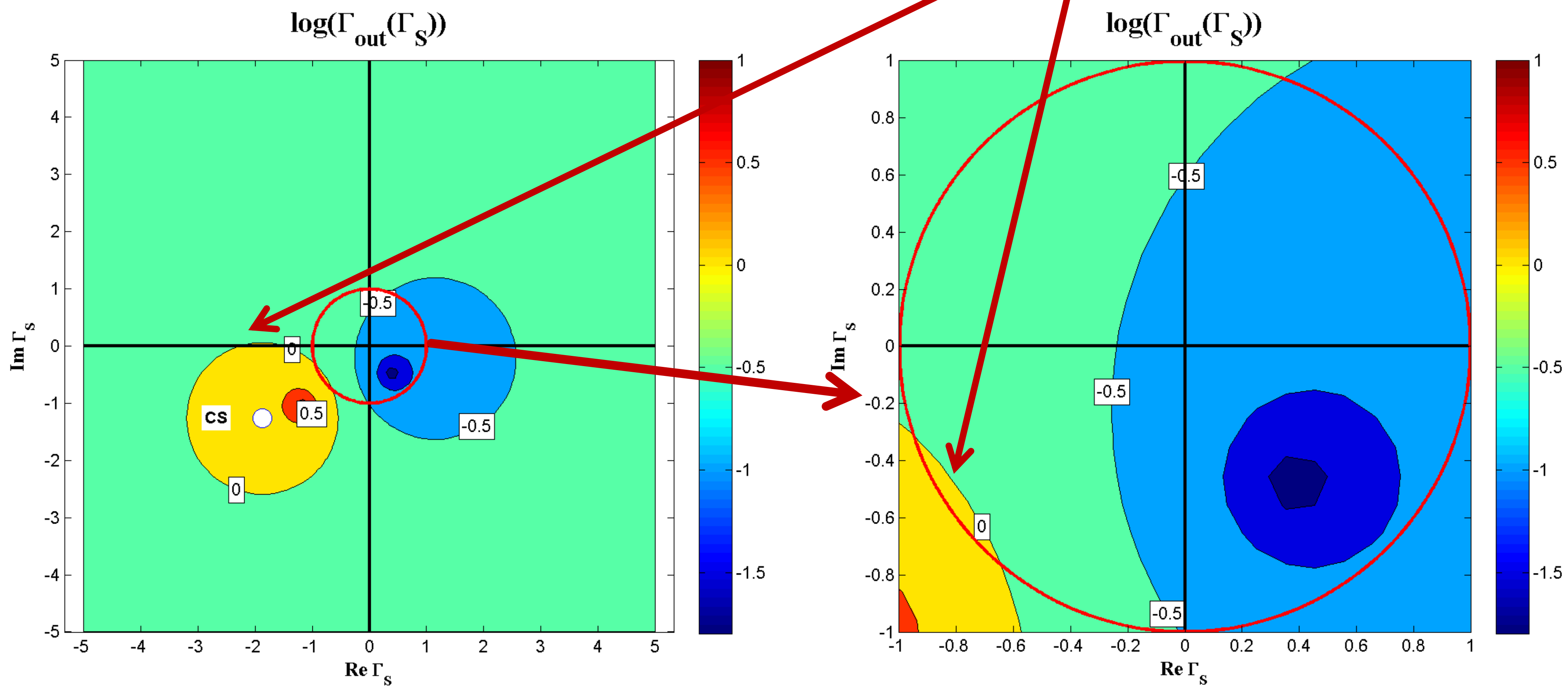
- $\log_{10}|\Gamma_{in}| = 0$, Γ_L, CSOUT



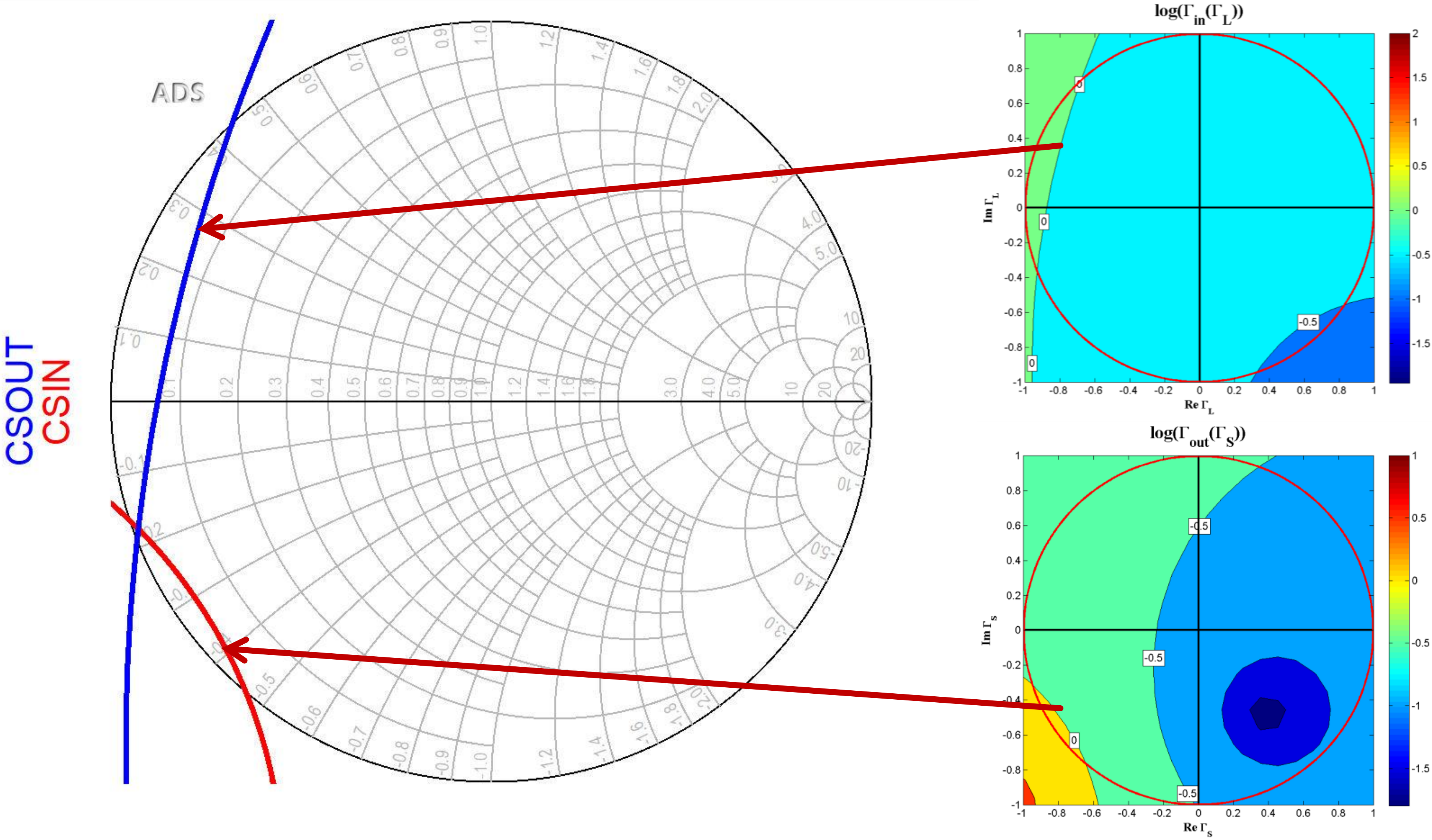
Contour lines of $\log_{10}|\Gamma_{out}|$

- $\log_{10}|\Gamma_{out}| = 0$, Γ_S, CSIN

$\log_{10}|\Gamma_{out}| = 0$,
$|\Gamma_{out}| = 1$



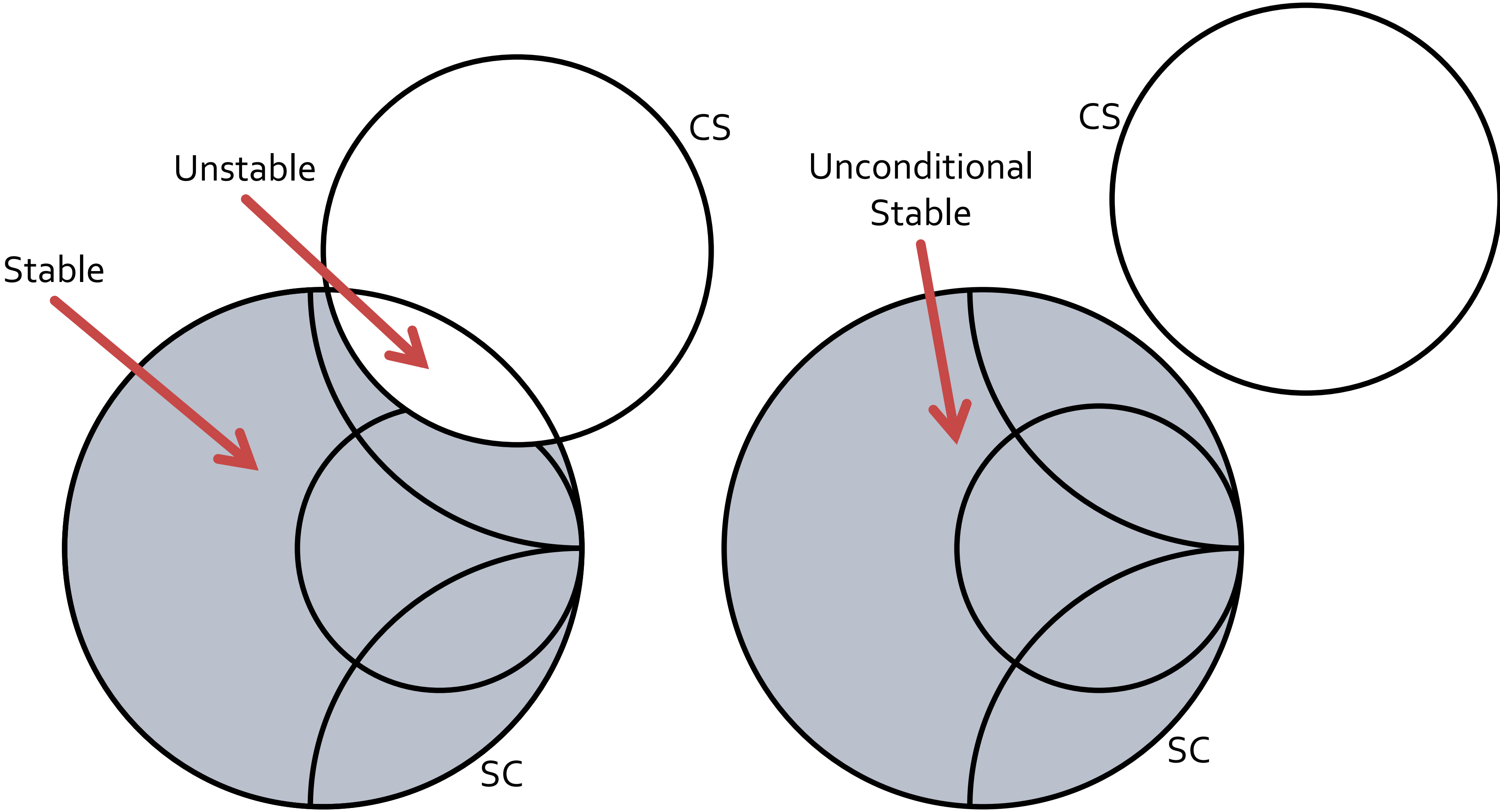
CSIN, CSOUT



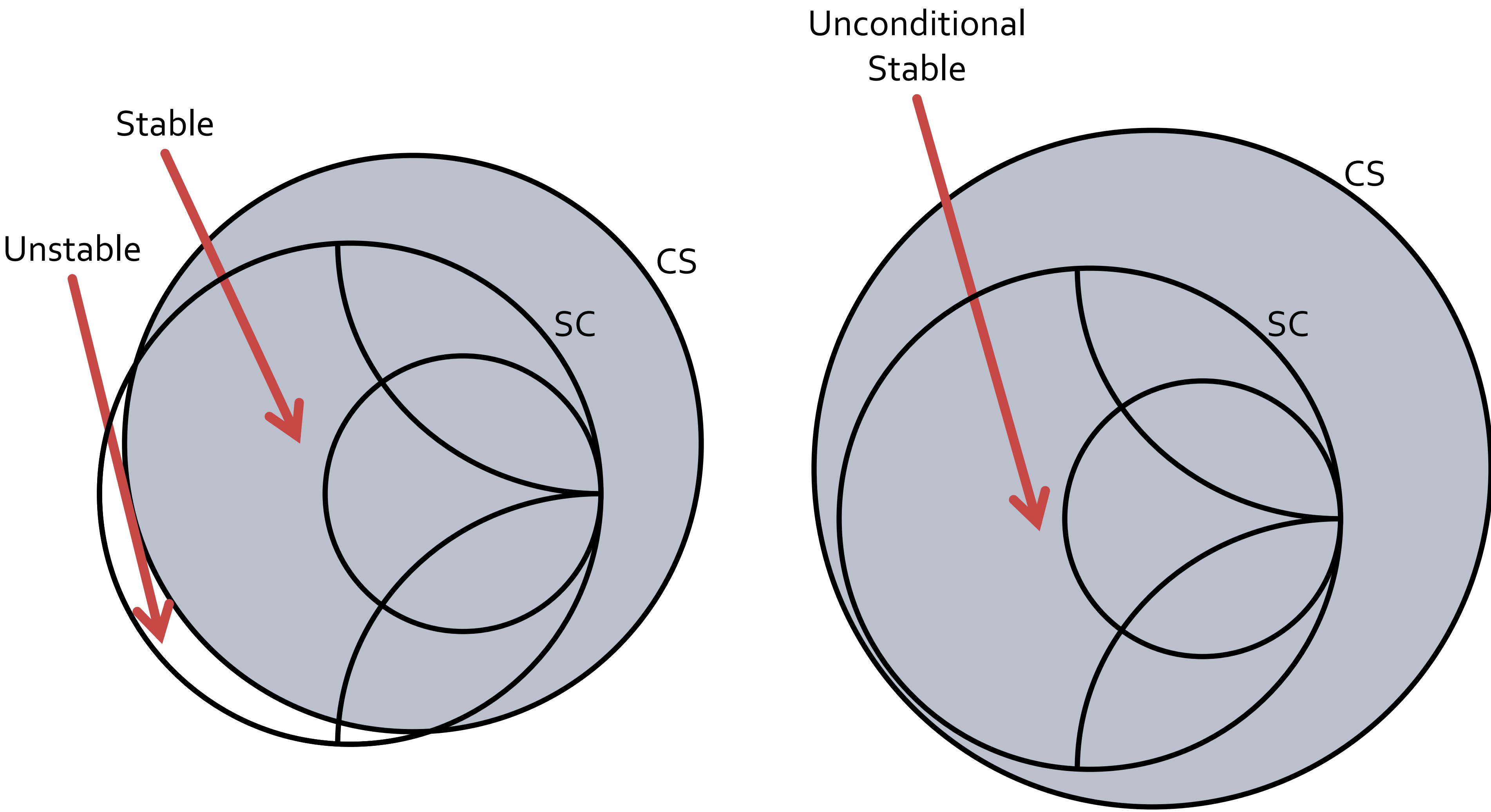
Continued

Amplifiers - Stability

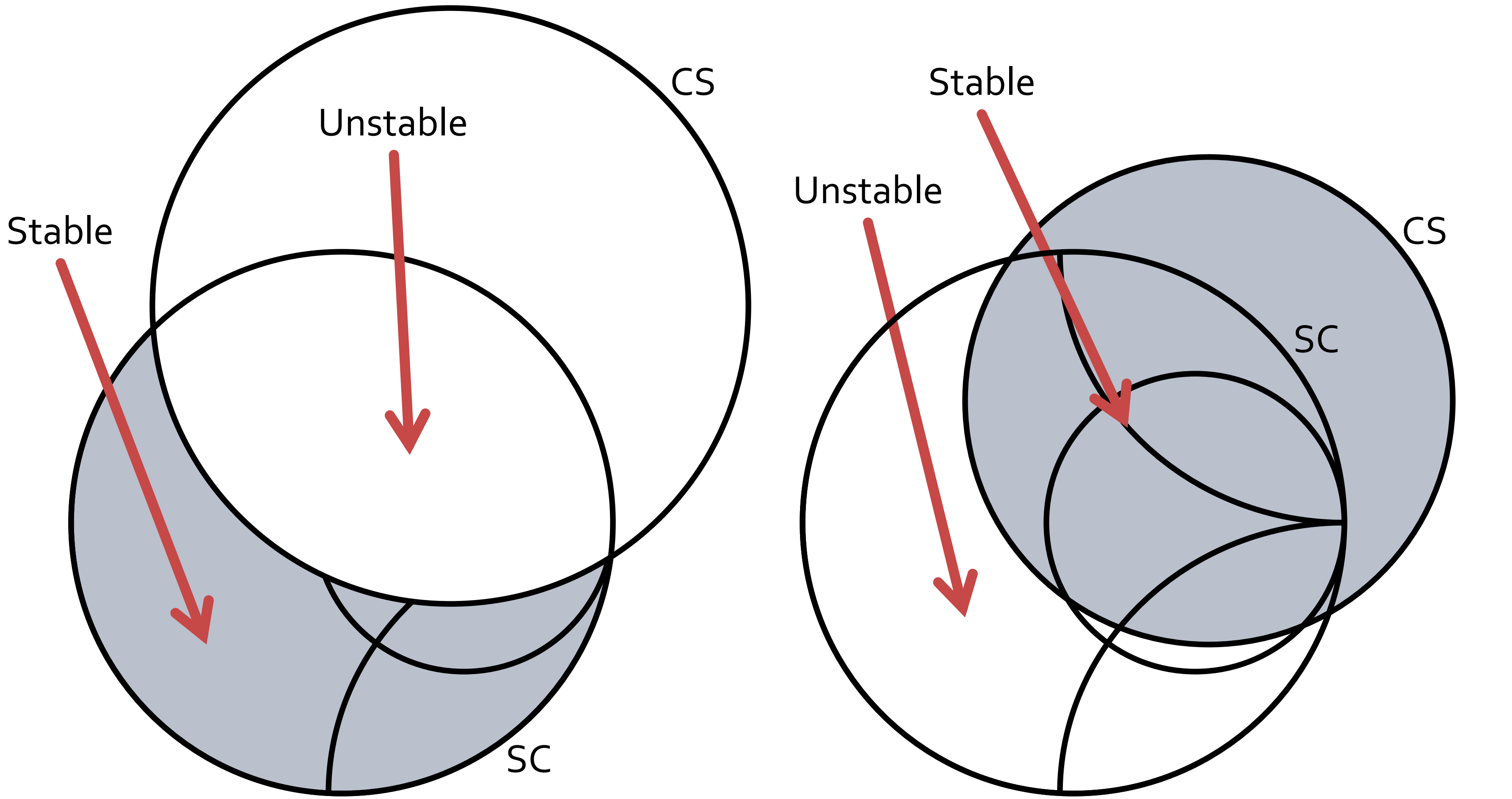
Several possible positioning



Several possible positioning



(Quite) Rare positioning



Stability

- **Unconditional stability:** the circuit is unconditionally stable if $|\Gamma_{in}|<1$ and $|\Gamma_{out}|<1$ for **any** passive impedance of the load/source
- **Conditional stability:** the circuit is conditionally stable if $|\Gamma_{in}|<1$ and $|\Gamma_{out}|<1$ only for **some** passive impedance of the load/source

Unconditional stability

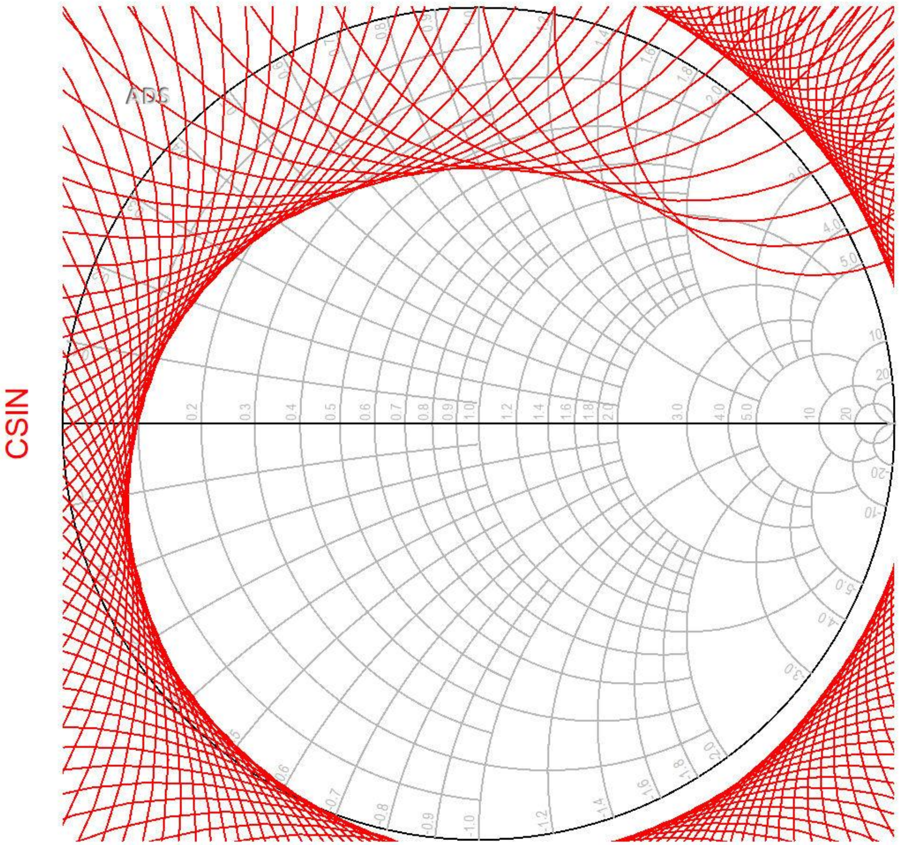
- The two-port is unconditionally stable if:
  - The stability circle is disjoint with the Smith Chart (exterior to the Chart) and the stable region is outside the circle
  - The stability circle encloses the entire Smith Chart and the stable region is inside the circle
- One mandatory condition for unconditional stability is |S11| < 1 (CSOUT) or |S22| < 1 (CSIN) – if in at least one point the two-port is not stable then it cannot be unconditionally stable
- Mathematically :

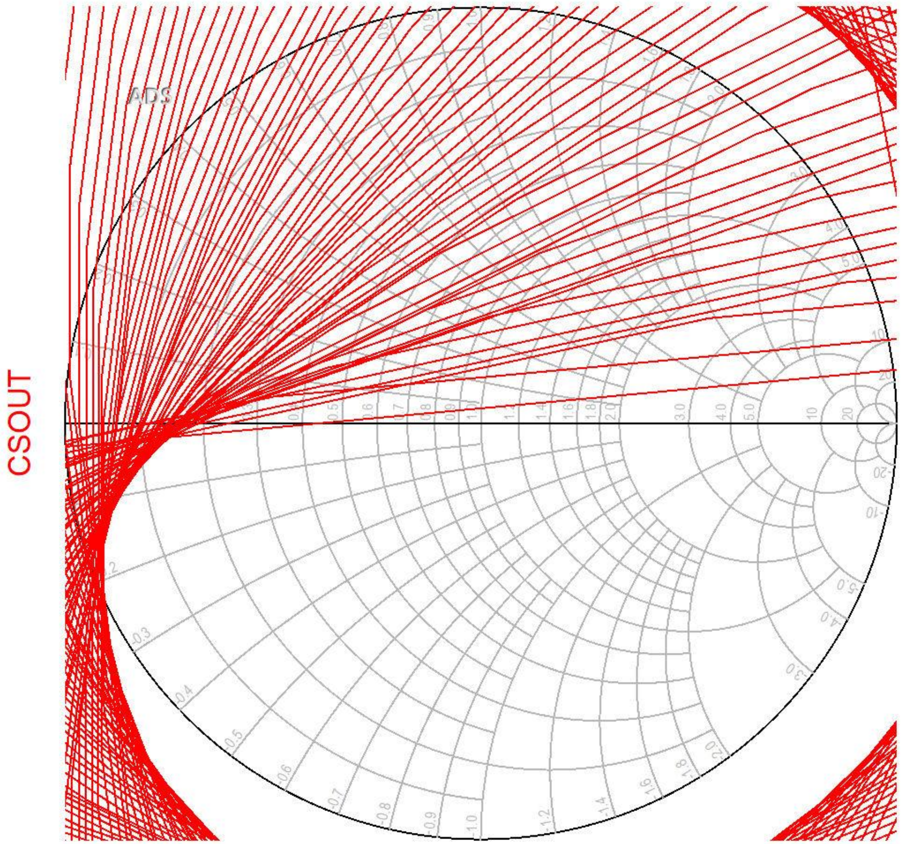
$$\begin{cases} \left| |C_L| - R_L \right| > 1 \\ |S_{11}| < 1 \end{cases} \qquad \begin{cases} \left| |C_S| - R_S \right| > 1 \\ |S_{22}| < 1 \end{cases}$$

Tests for Unconditional Stability

- Useful for wide frequency range analysis
- It is not enough to check the stability only at the operating frequencies
  - we must obtain stable operation for chosen Γ_L and Γ_S at **any** frequency

Circles in wide frequency range





Rollet's condition

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2 \cdot |S_{12} \cdot S_{21}|} \qquad \Delta = S_{11} \cdot S_{22} - S_{12} \cdot S_{21}$$

- The two-port is **unconditionally stable** if:
- two conditions are simultaneously satisfied:
  - K > 1
  - |Δ| < 1
- together with the implicit conditions:
  - |S11| < 1
  - |S22| < 1

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2 \cdot |S_{12} \cdot S_{21}|} > 1 \qquad |\Delta| = |S_{11} \cdot S_{22} - S_{12} \cdot S_{21}| < 1$$

μ Criterion

- Rollet's condition cannot be used to compare the relative stability of two or more devices because it involves constraints on two separate parameters, K and Δ

$$\mu = \frac{1-|S_{11}|^2}{|S_{22} - \Delta \cdot S_{11}^*| + |S_{12} \cdot S_{21}|} > 1$$

- The two-port is **unconditionally stable** if:
  - $\mu > 1$
- together with the implicit conditions:
  - $|S_{11}| < 1$
  - $|S_{22}| < 1$
- In addition, it can be said that larger values of μ imply greater stability
  - μ is the distance from the center of the Smith Chart to the closest output stability circle

μ′ Criterion

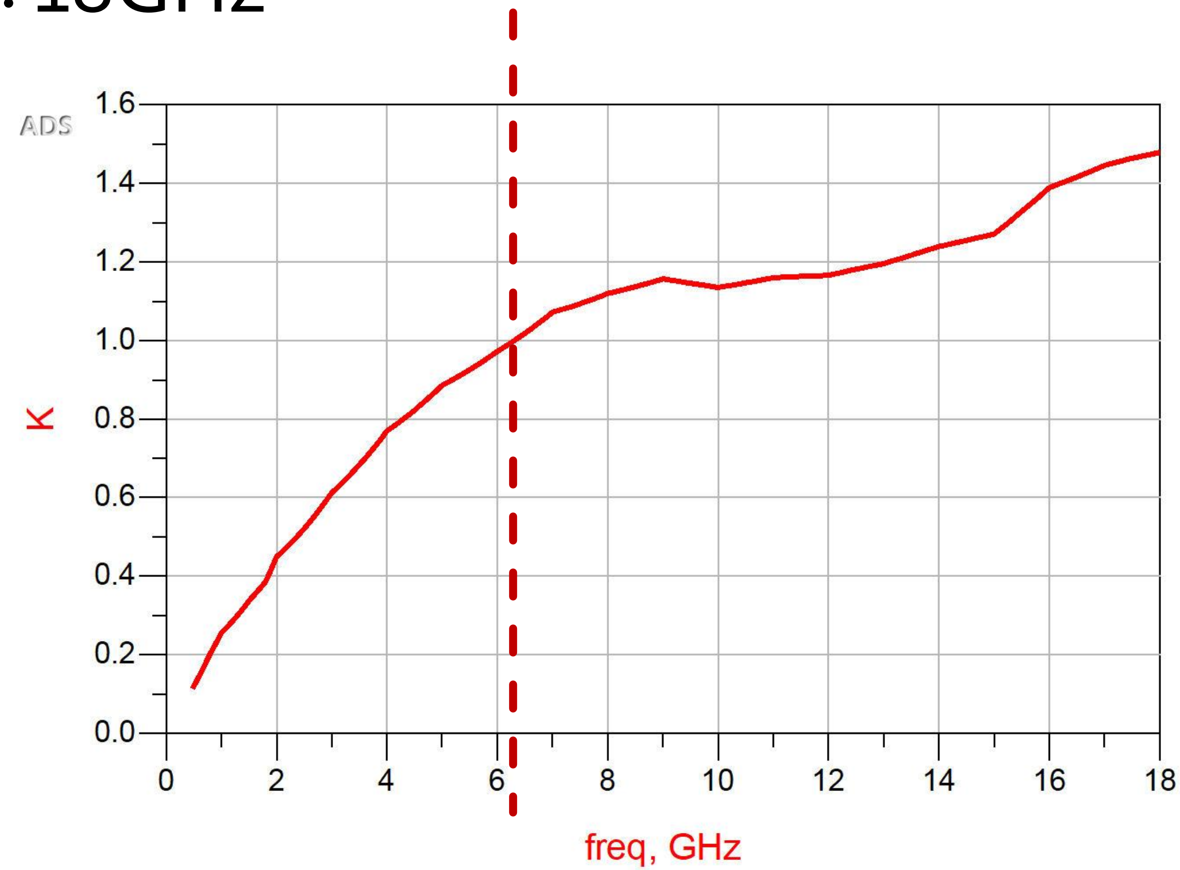
- Dual parameter to μ, determined in relation to the input stability circles

$$\mu' = \frac{1-|S_{22}|^2}{|S_{11}-\Delta\cdot S_{22}^*|+|S_{12}\cdot S_{21}|} > 1$$

- The two-port is **unconditionally stable** if:
  - $\mu' > 1$
- together with the implicit conditions:
  - $|S_{11}| < 1$
  - $|S_{22}| < 1$
- In addition, it can be said that larger values of μ′ imply greater stability
  - μ′ is the distance from the center of the Smith Chart to the closest input stability circle

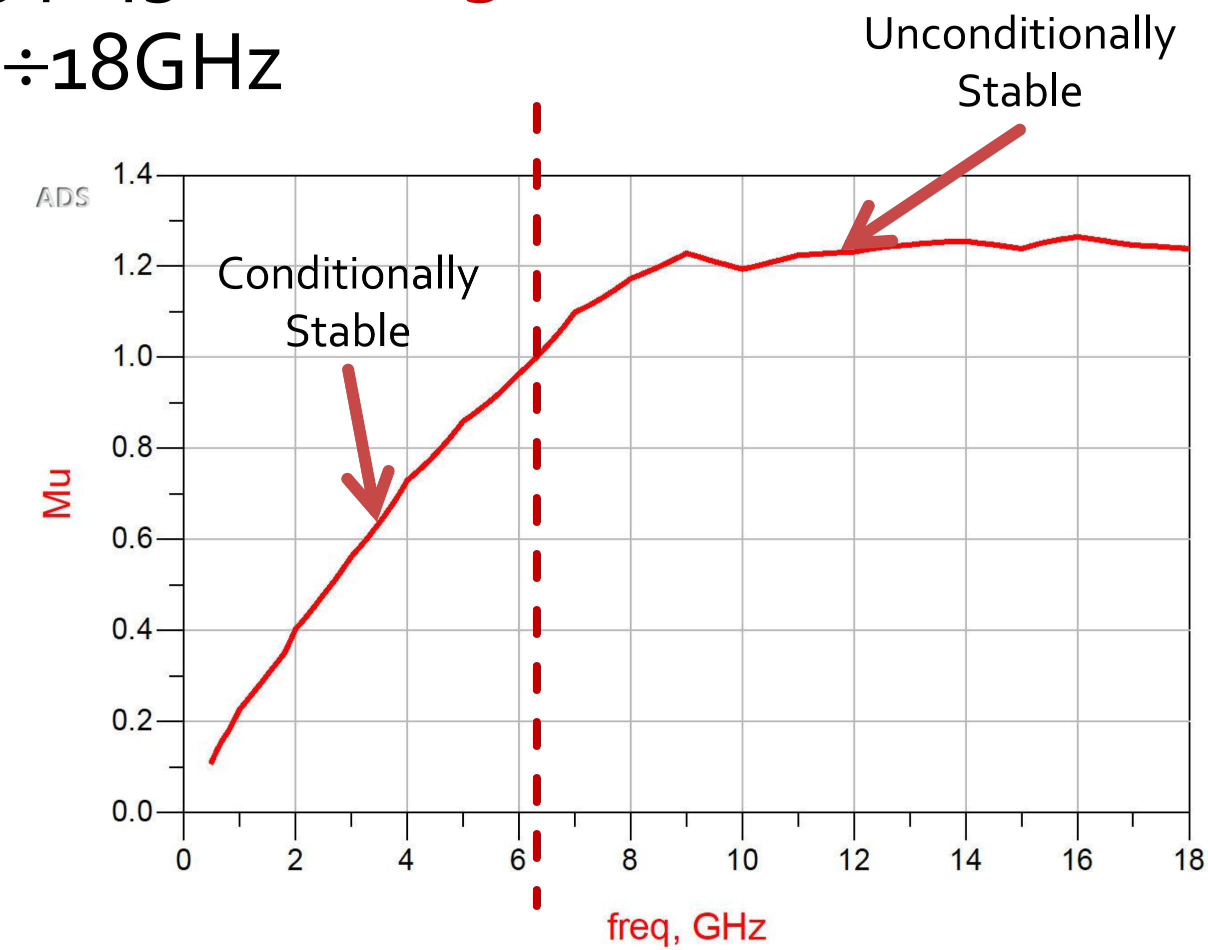
Rollet's condition

- ATF-34143 **at Vds=3V Id=20mA**.
- @0.5÷18GHz



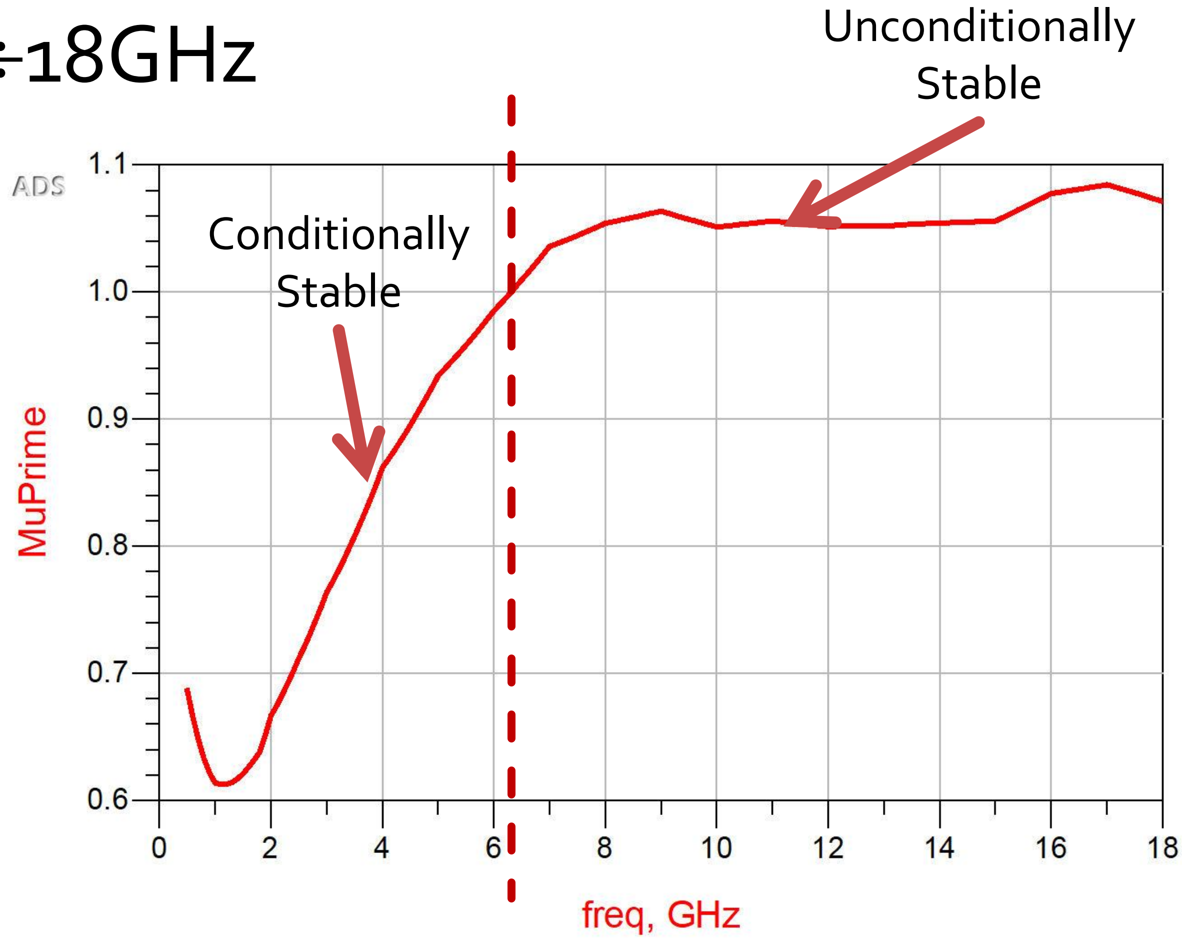
μ Criterion

- ATF-34143 **at Vds=3V Id=20mA**.
- @0.5÷18GHz



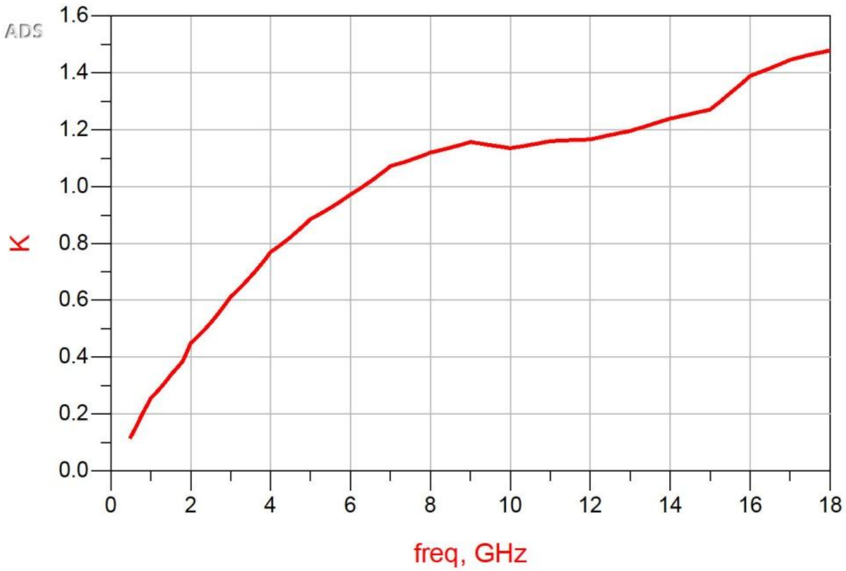
μ' Criterion

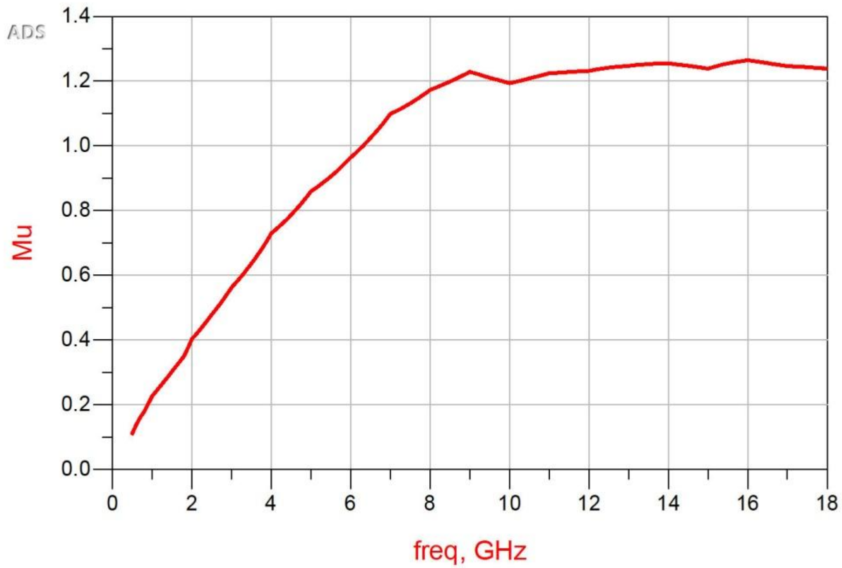
- ATF-34143 **at Vds=3V Id=20mA**.
- @0.5÷18GHz



Stability

- ATF-34143 **at Vds=3V Id=20mA**.
- @0.5÷18GHz
- unconditionally stable for f > 6.31GHz

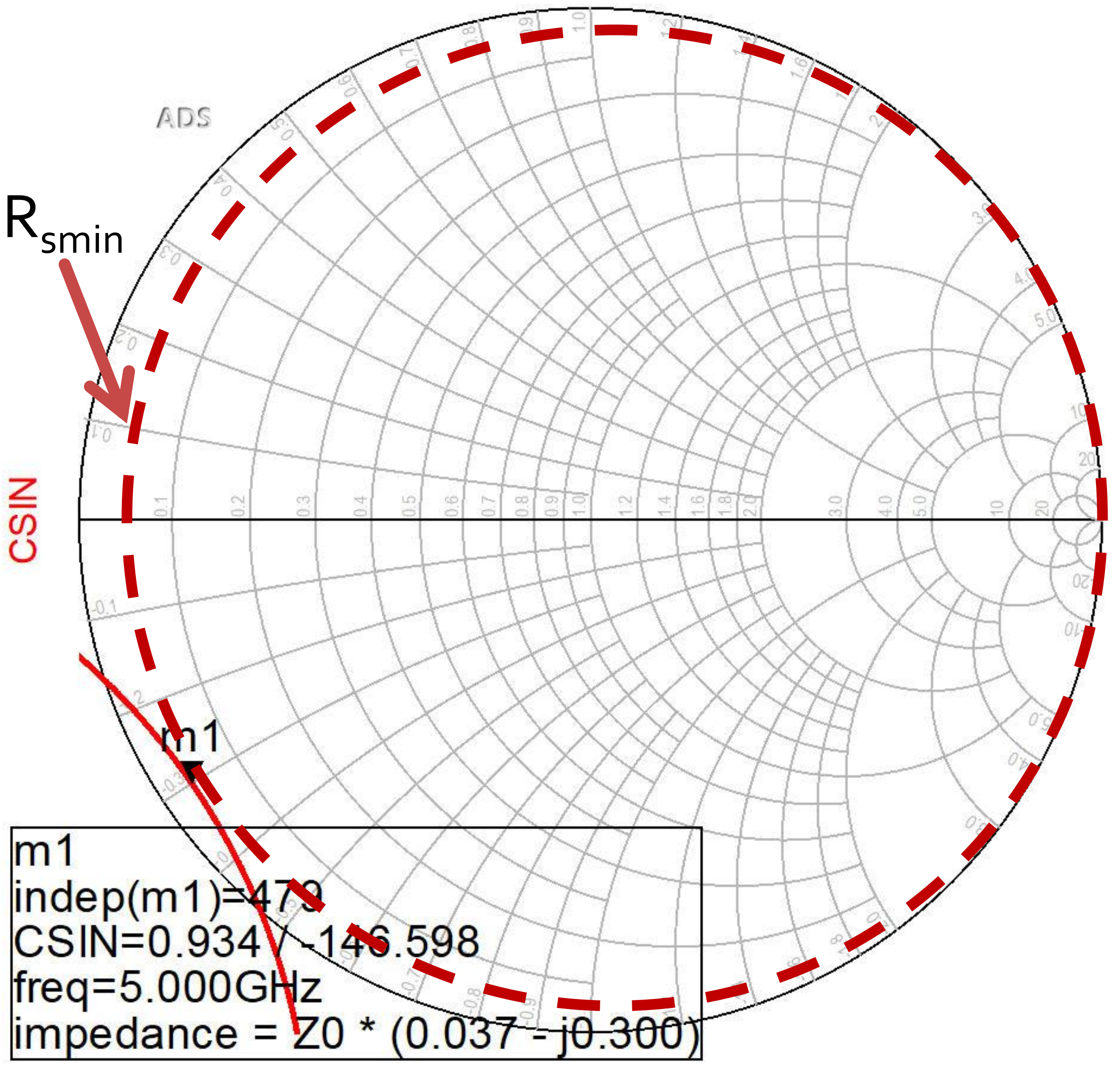


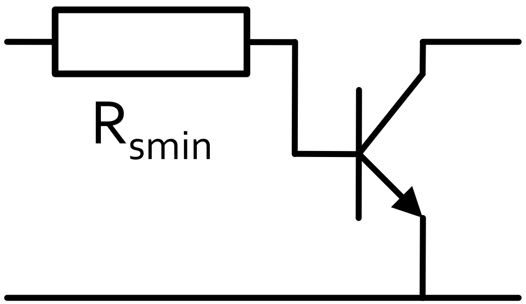


Stabilization of two-port

- Unconditional stability in a wide frequency range has some important advantages
  - Ex: We can use ATF 34143 to design a (conditionally) stable amplifier at 5GHz, but this design is useless if the amplifier oscillates at 500MHz ($\mu \approx 0.1$)
- **The minimal requirement** when working with conditionally stable devices is to **check stability** at several frequencies over the operating bandwidth and outside the bandwidth
- Unconditional stability can be forced by inserting series/shunt resistors at two-port’s input/output (with loss of gain!)

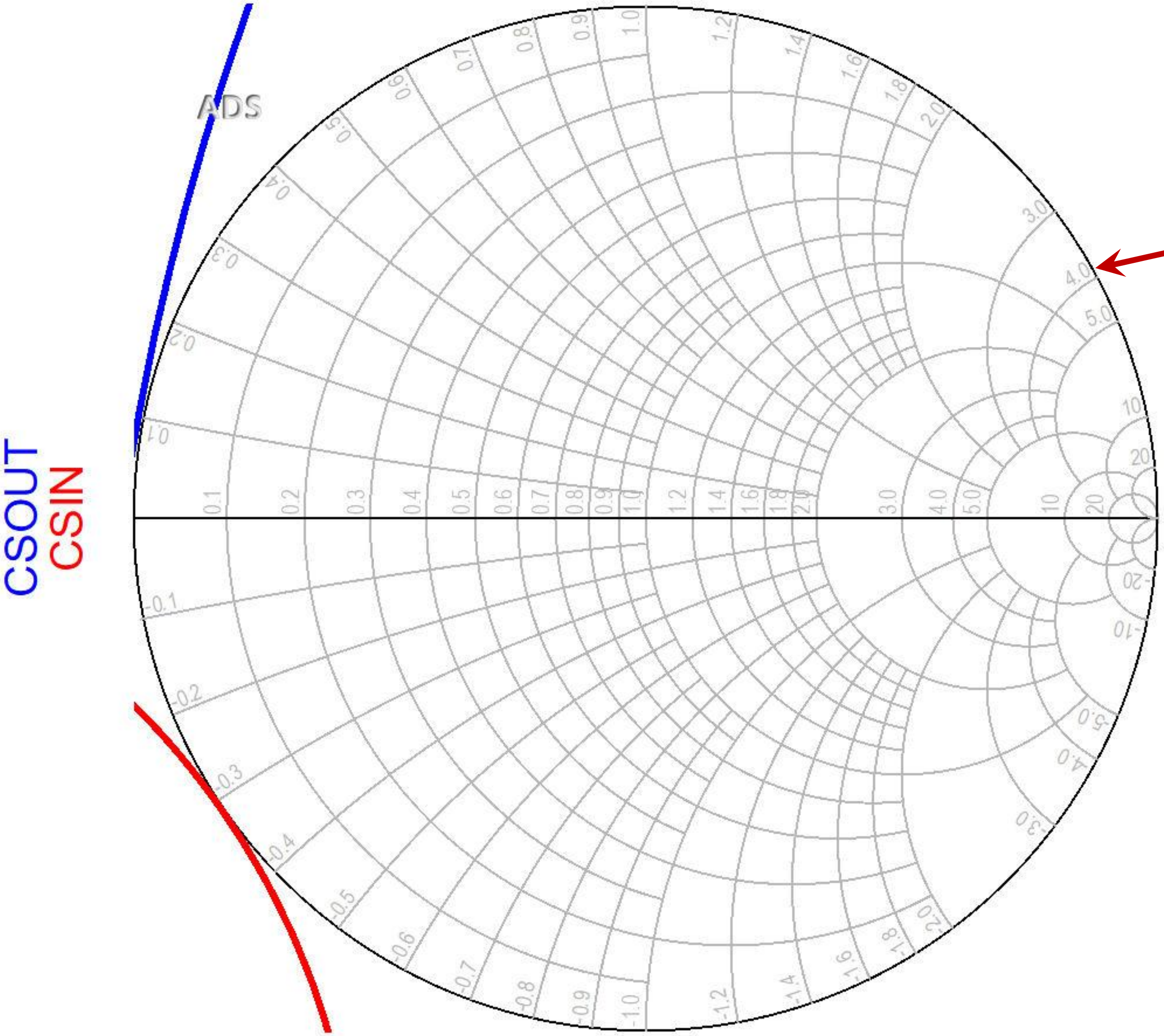
Input series resistor

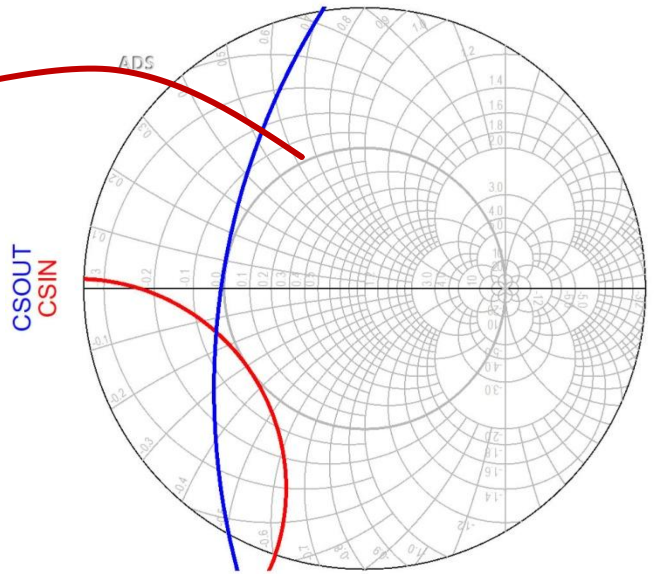




$$R_{s\min} = 0.037 \cdot 50\Omega = 1.85\Omega$$

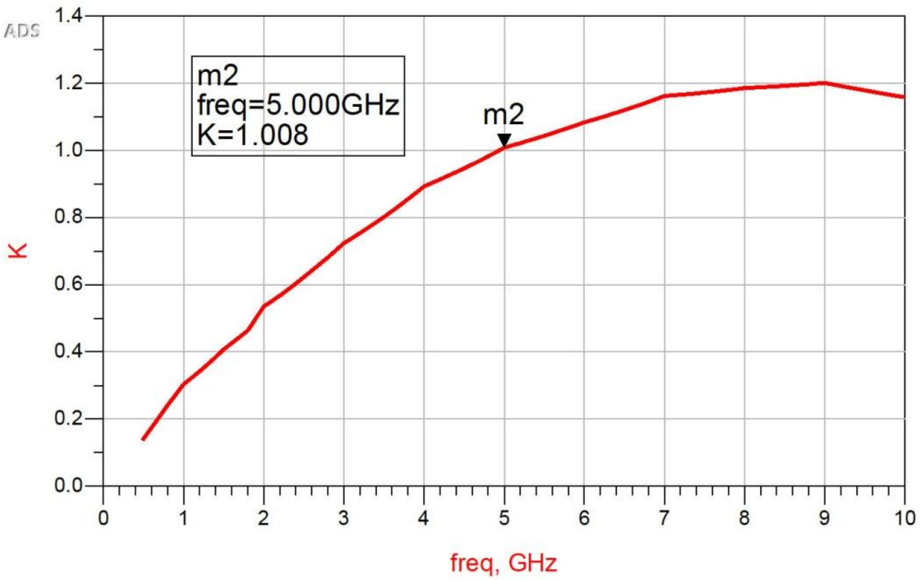
ADS, Rs = 2Ω

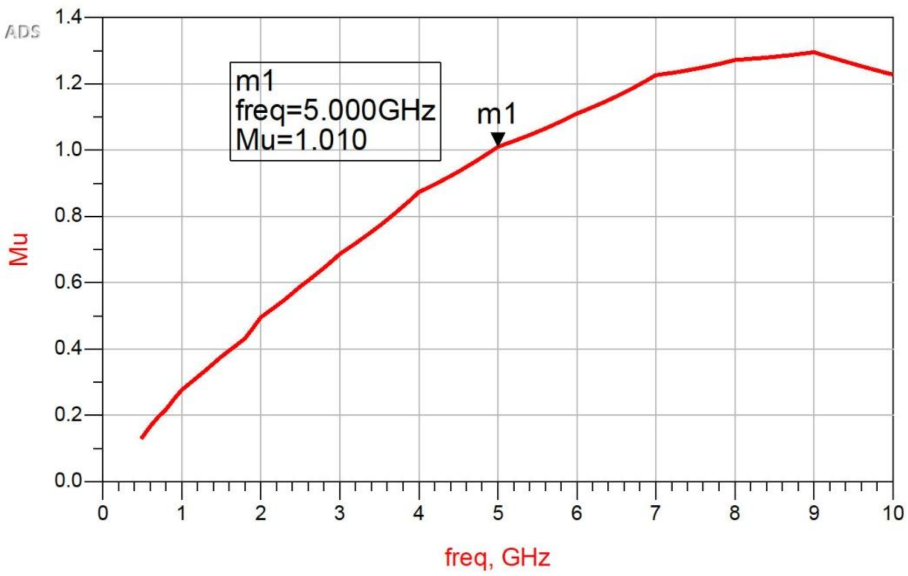




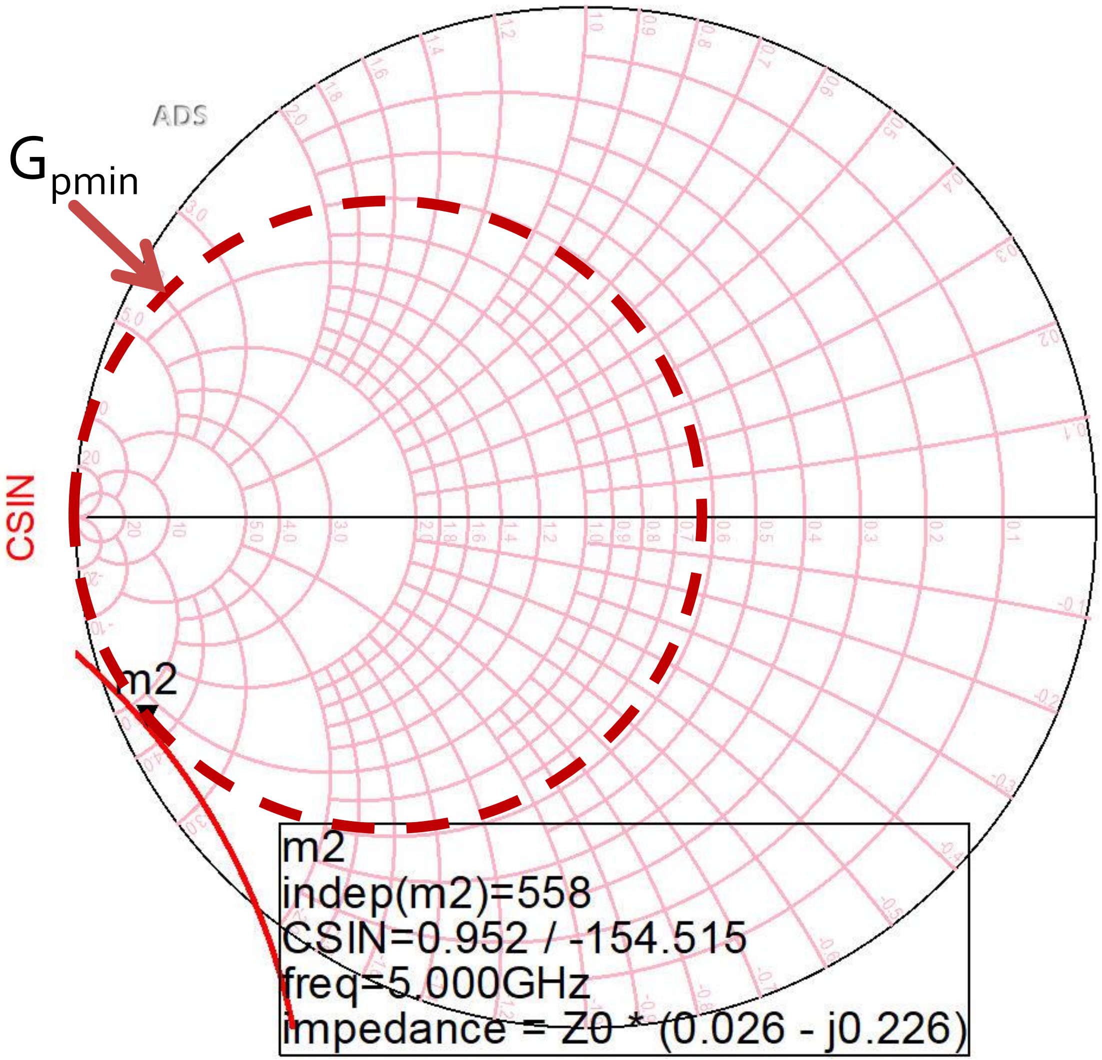
Input series resistor

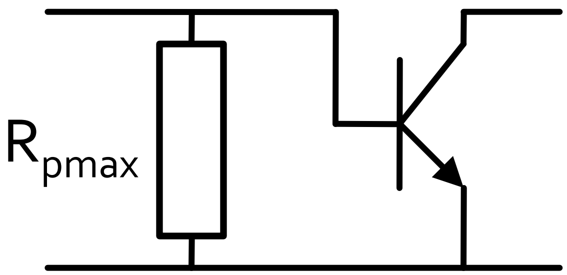
- Rs = 2Ω
- K = 1.008, MAG = 13.694dB @ 5GHz
  - no stabilization, K = 0.886, MAG = 14.248dB @ 5GHz





Input shunt resistor



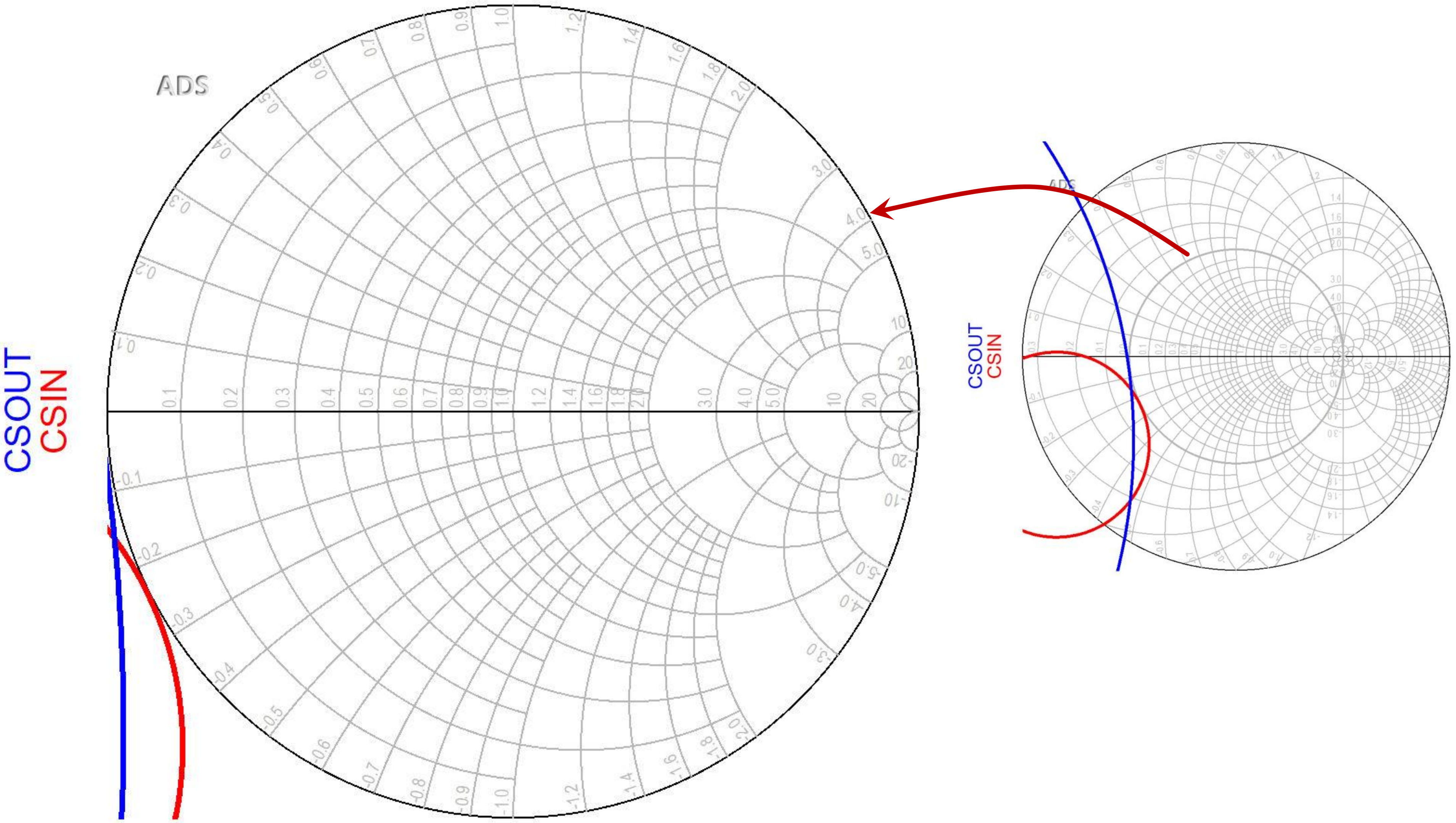


$$R_{p\max} = \frac{1}{G_{p\min}}$$

$$\frac{1}{0.026 - j \cdot 0.226} = 0.502 + j \cdot 4.367$$

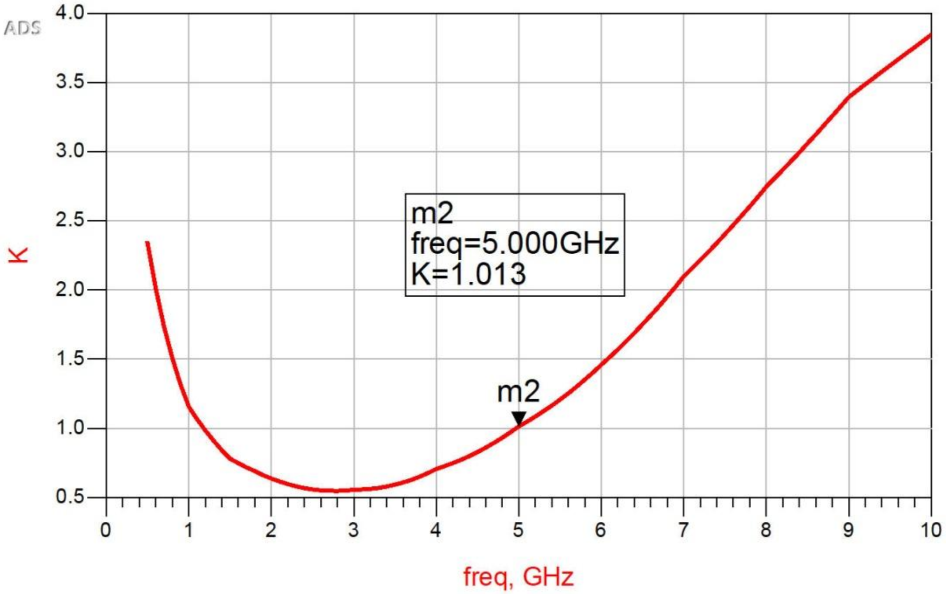
$$R_{p\max} = \frac{50\Omega}{0.502} = 99.6\Omega$$

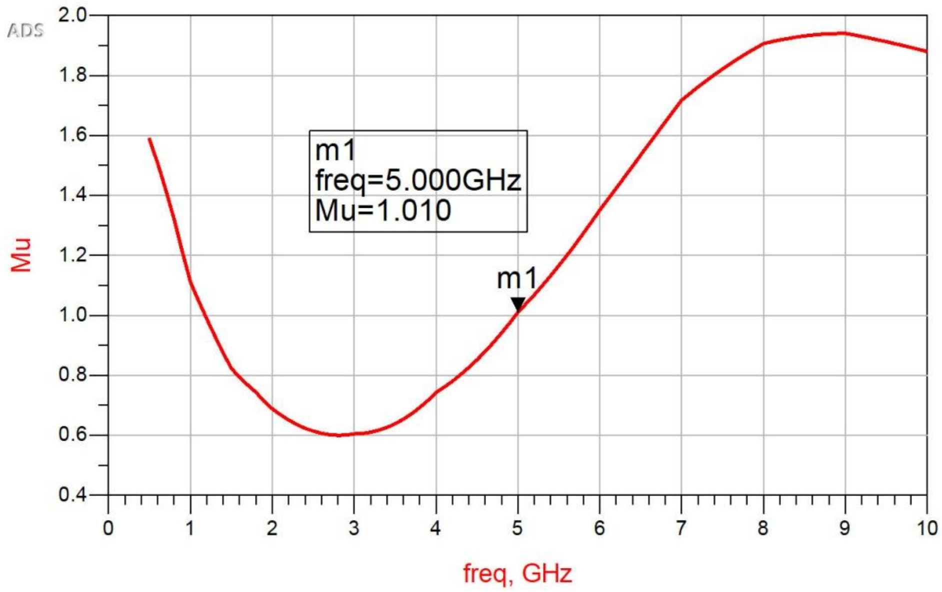
ADS, Rp = 90Ω



Input shunt resistor

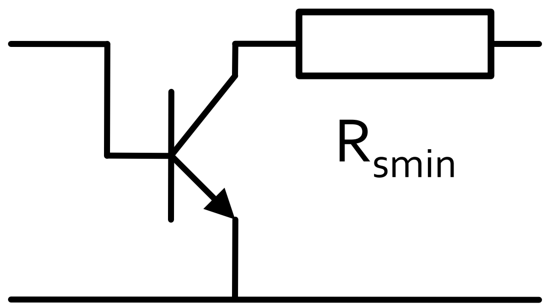
- Rp = 90Ω
- K = 1.013, MAG = 13.561dB @ 5GHz
  - no stabilization, K = 0.886, MAG = 14.248dB @ 5GHz

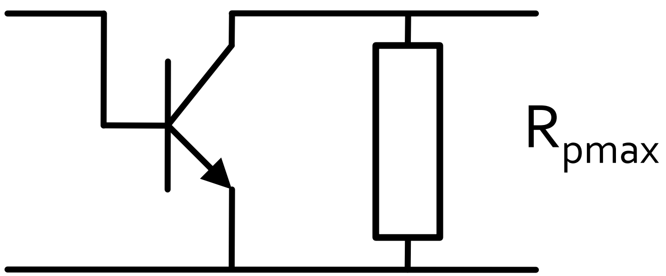




Output series/shunt resistor

- The procedure can be applied similarly at the output (finding g/r circles tangent to CSOUT)
- From previous examples, resistive loading at the input has a positive effect over output stability and vice versa (resistive loading at the output, effect over input stability)

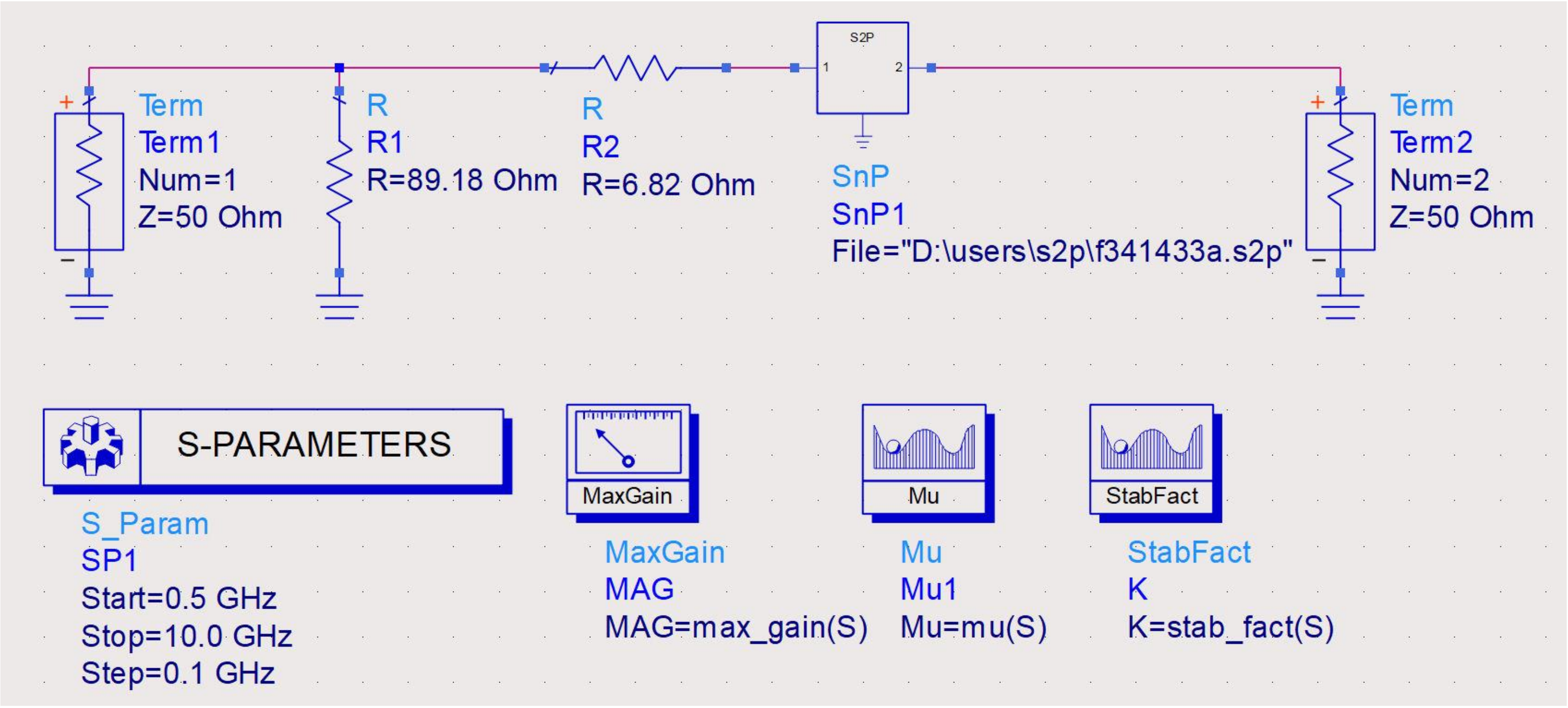




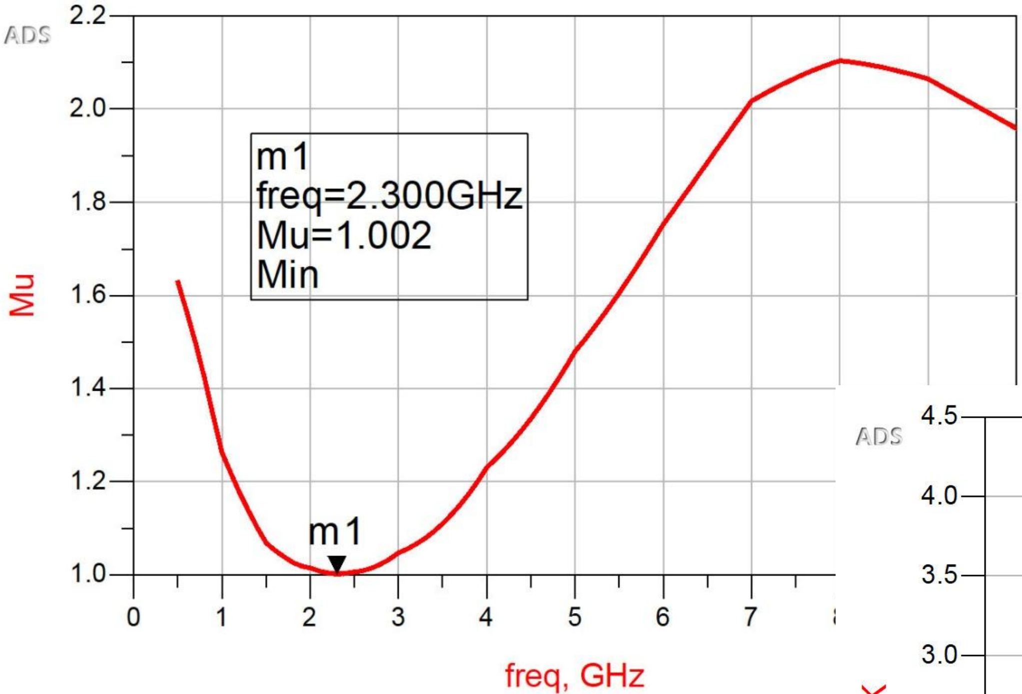
Stabilization of two-port

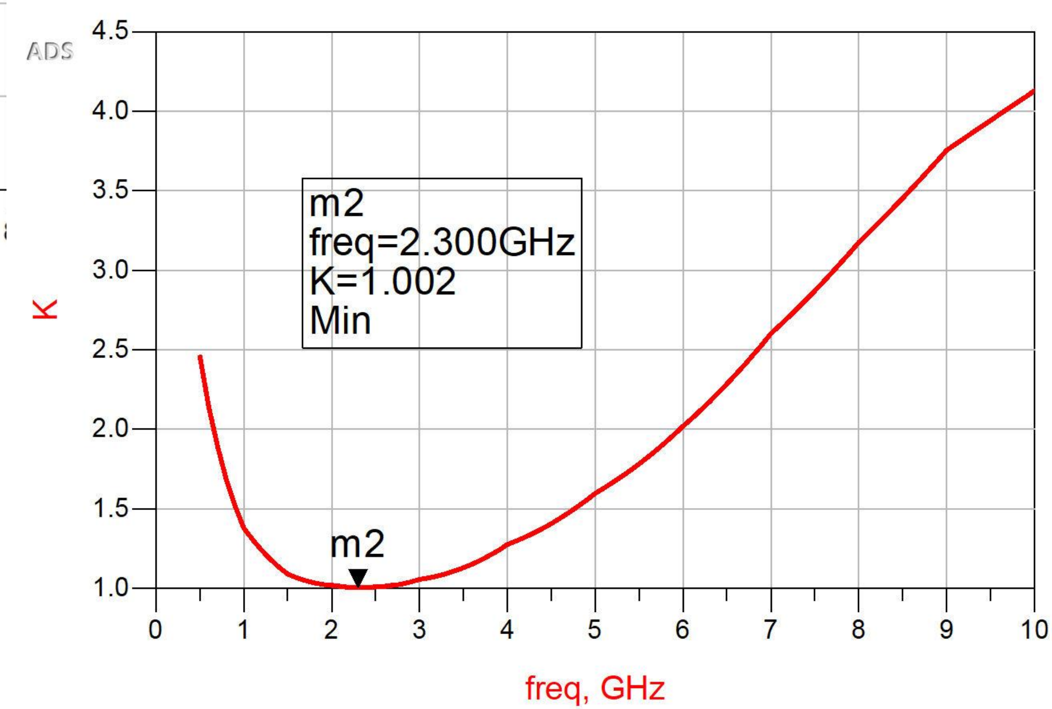
- Negative effect over the power gain
  - we must check MAG/MSG while designing resistive loading
- Negative effect over the noise (next lectures)
- We can choose one of the 4 possibilities or a combination which offers better results (depending on transistor, application etc.)
- We can use frequency selective loading
  - Ex: RL, RC circuits which sacrifice performance only when needed to improve stability and have no effect at frequencies where the device is already stable
- It might be possible (and should be checked) that stability is improved as an effect of parasitic elements of biasing circuits (bypass capacitors and RF chokes)

Stabilization of two-port

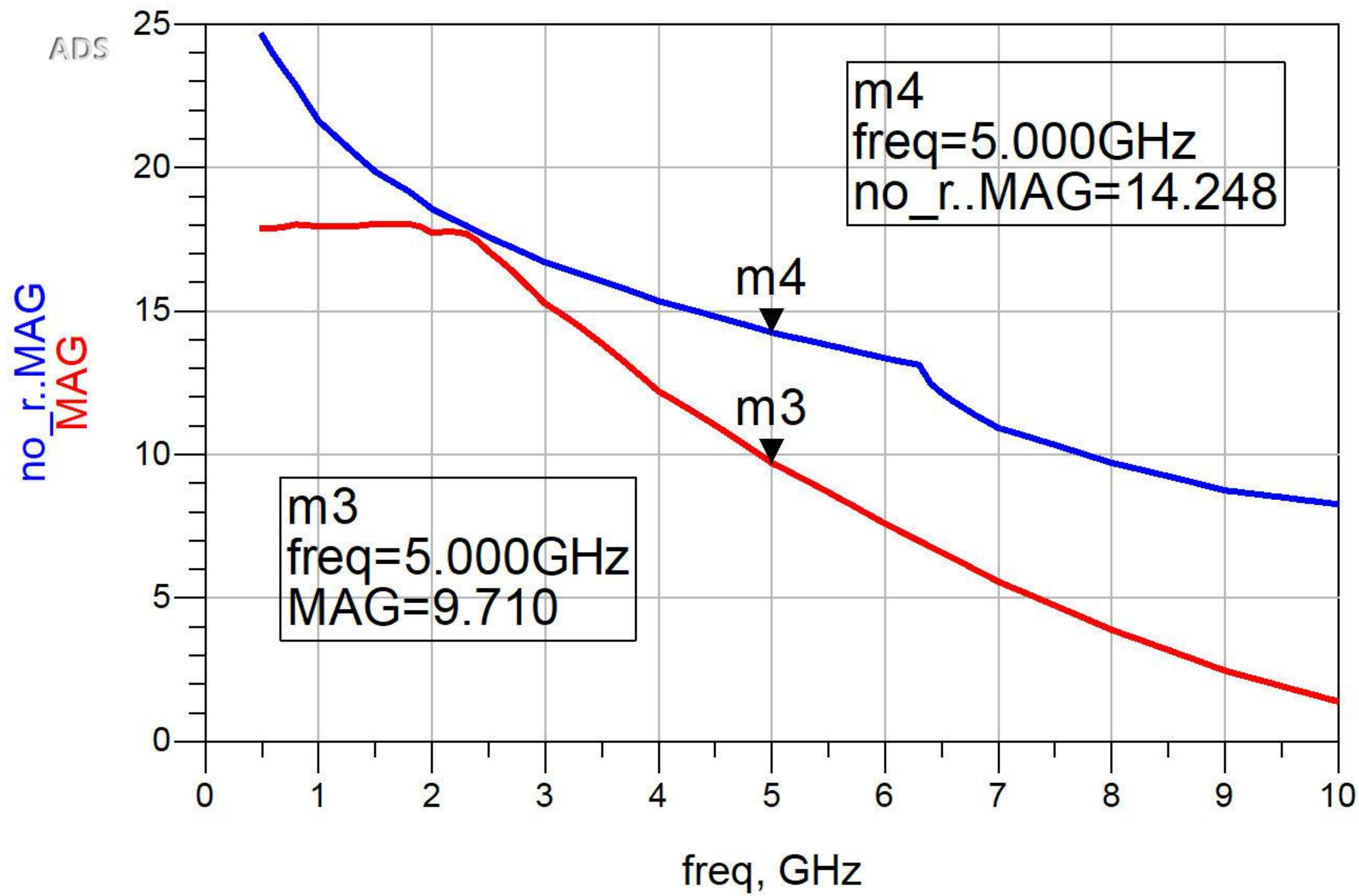


Stabilization of two-port

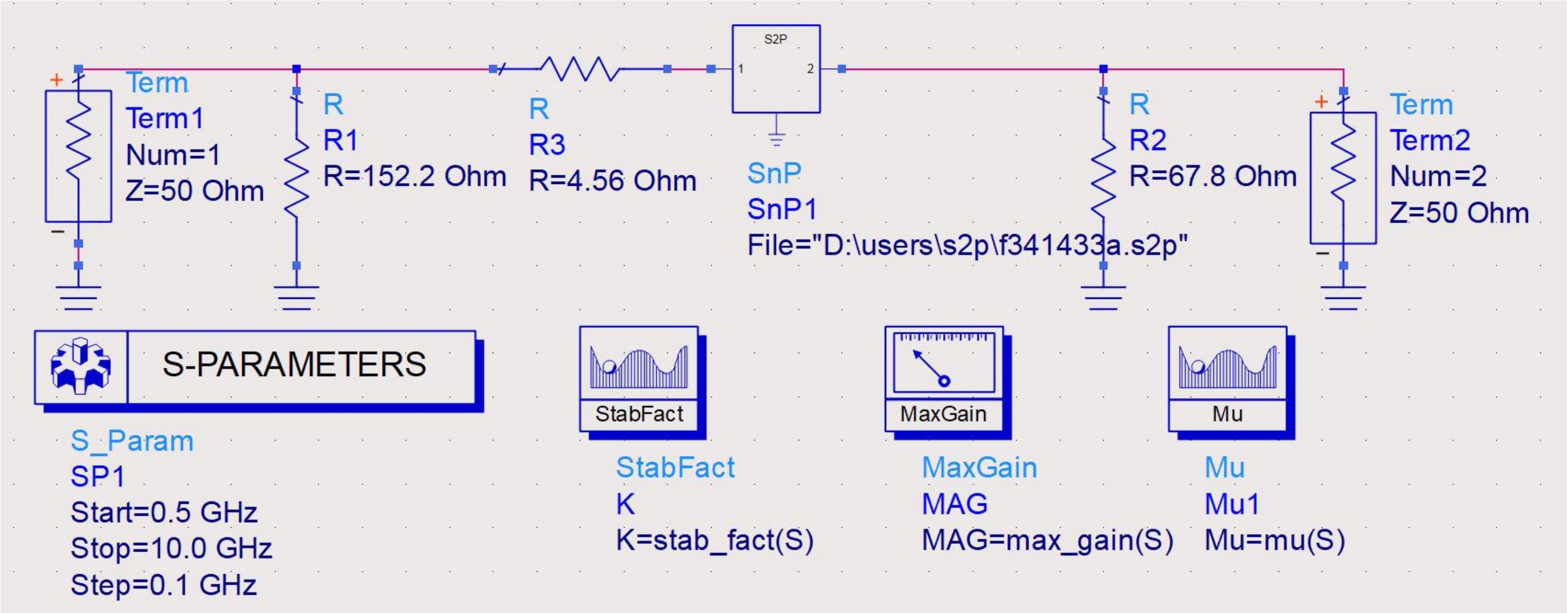




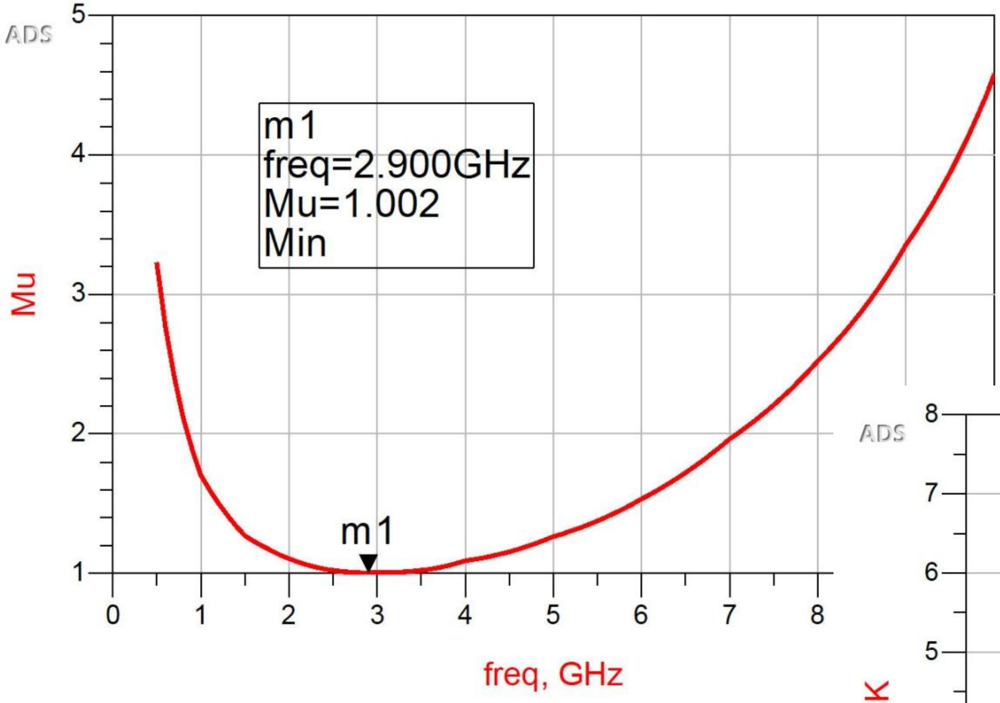
Stabilization of two-port

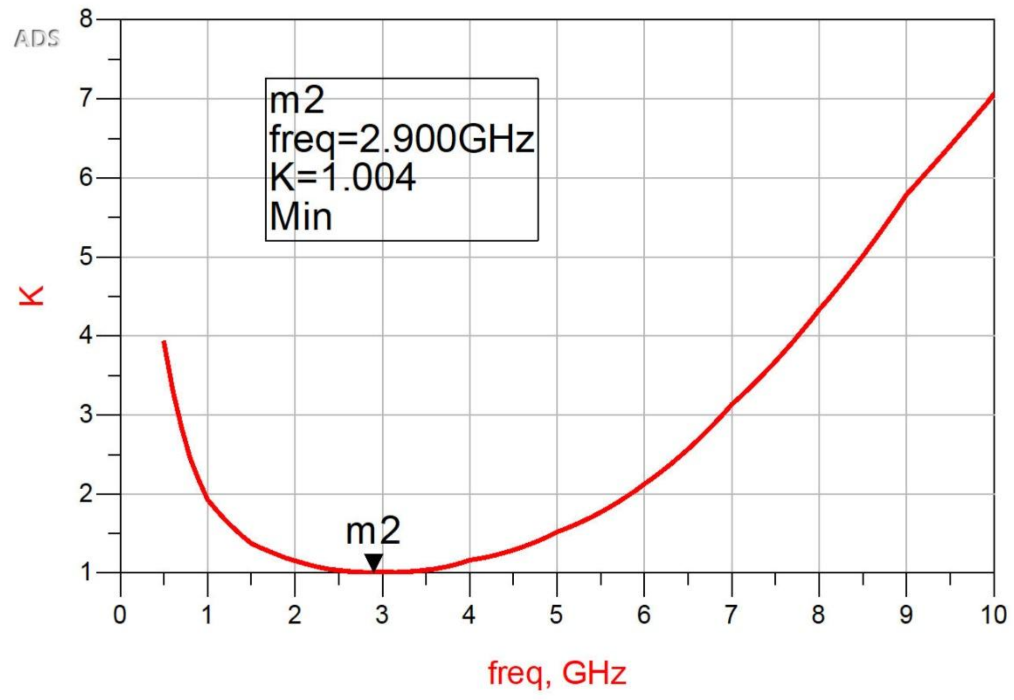


Stabilization of two-port

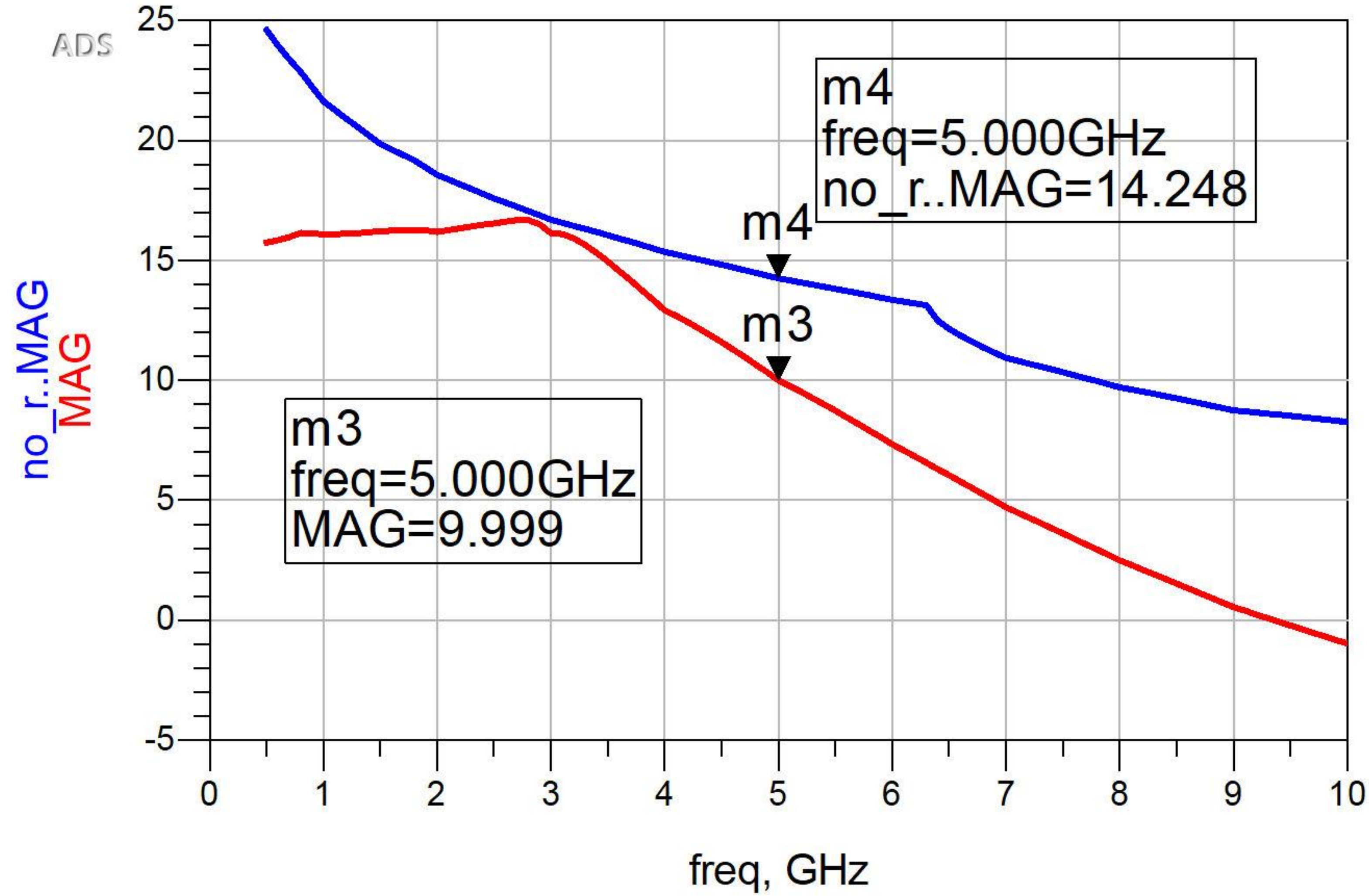


Stabilization of two-port





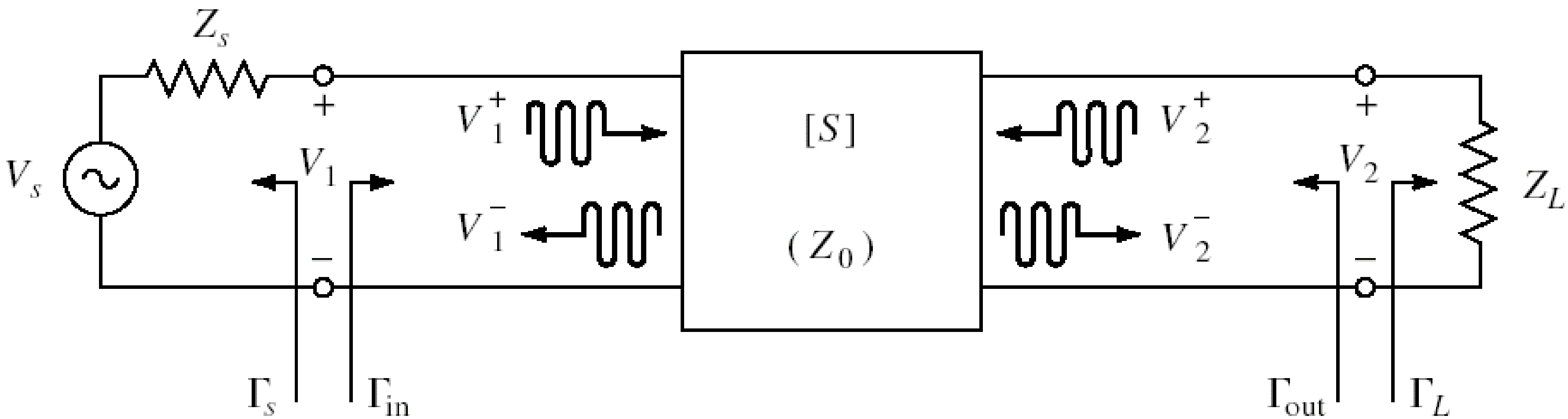
Stabilization of two-port



Power Gain of Microwave Amplifiers

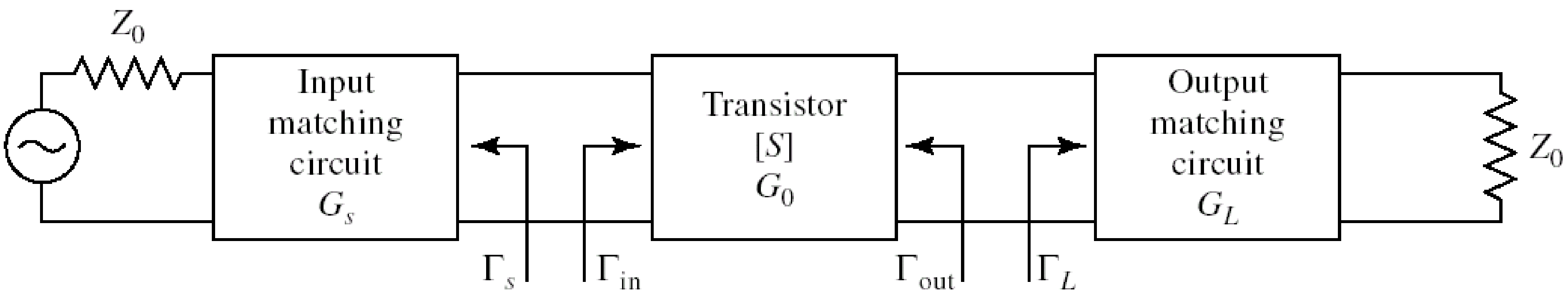
Microwave Amplifiers

Amplifier as two-port



- For an amplifier two-port we are interested in:
  - stability
  - **power gain**
  - noise (sometimes – small signals)
  - linearity (sometimes – large signals)

Design for Maximum Gain



- Maximum power gain (complex conjugate matching):

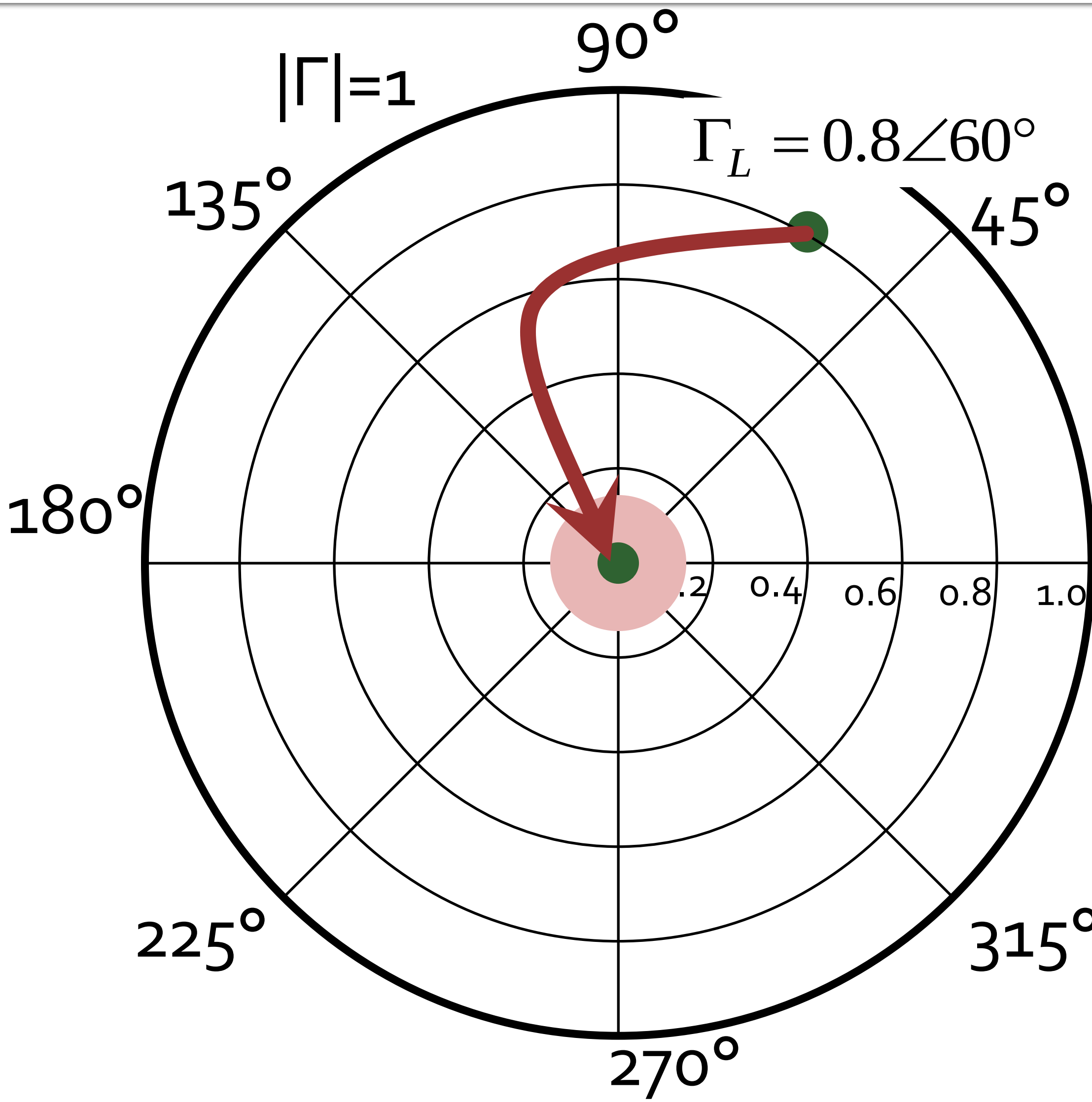
$$\Gamma_{in} = \Gamma_S^* \qquad \Gamma_{out} = \Gamma_L^*$$

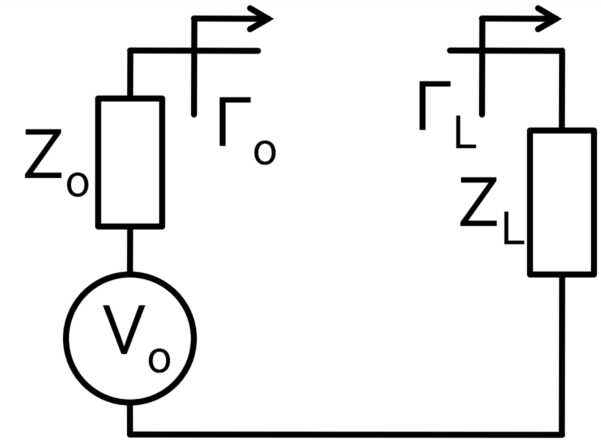
- For lossless matching sections

$$G_{T\max} = \frac{|S_{21}|^2 \cdot \left(1-|\Gamma_S|^2\right) \cdot \left(1-|\Gamma_L|^2\right)}{|1-\Gamma_S \cdot \Gamma_{in}|^2 \cdot |1-S_{22} \cdot \Gamma_L|^2} \qquad G_{T\max} = \frac{1}{1-|\Gamma_S|^2} \cdot |S_{21}|^2 \cdot \frac{1-|\Gamma_L|^2}{|1-S_{22} \cdot \Gamma_L|^2}$$

- For the general case of the bilateral transistor (S12≠0) Γ_{in} and Γ_{out} depend on each other so the input and output sections must be matched simultaneously

The Smith Chart, matching, $Z_L \neq Z_o$





Matching Z_L load to Z_o source.
We normalize Z_L over Z_o

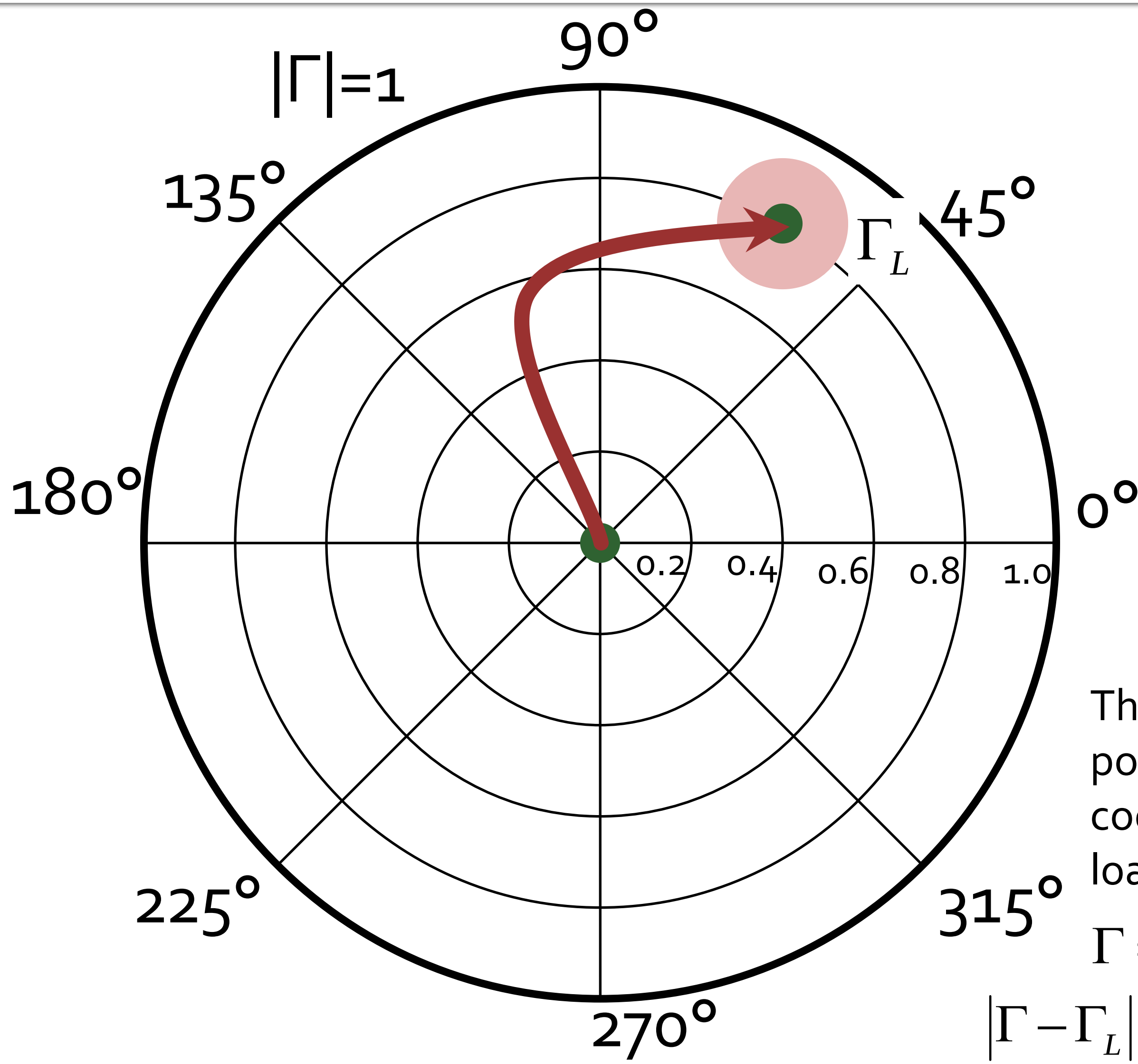
$$Z_L = 21.429\Omega + j\cdot 82.479\Omega$$
$$z_L = 0.429 + j\cdot 1.65$$
$$\Gamma_L = 0.8\angle 60°$$

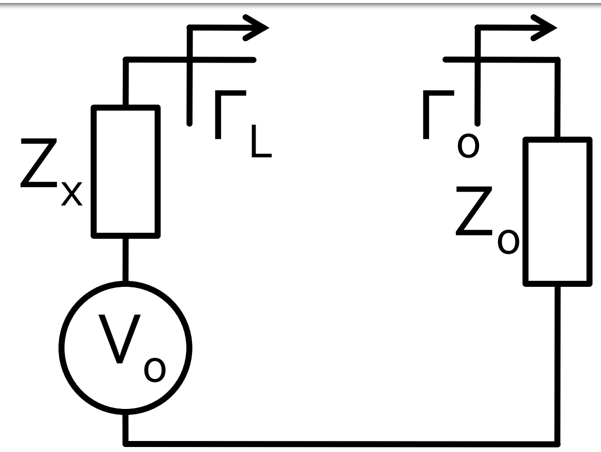
We must move the point denoting the reflection coefficient in the area where with a Z_o source we have:

$\Gamma_0 = 0$ perfect match

$|\Gamma_0| \leq \Gamma_m$ “good enough” match

The Smith Chart, matching, $Z_L = Z_o$





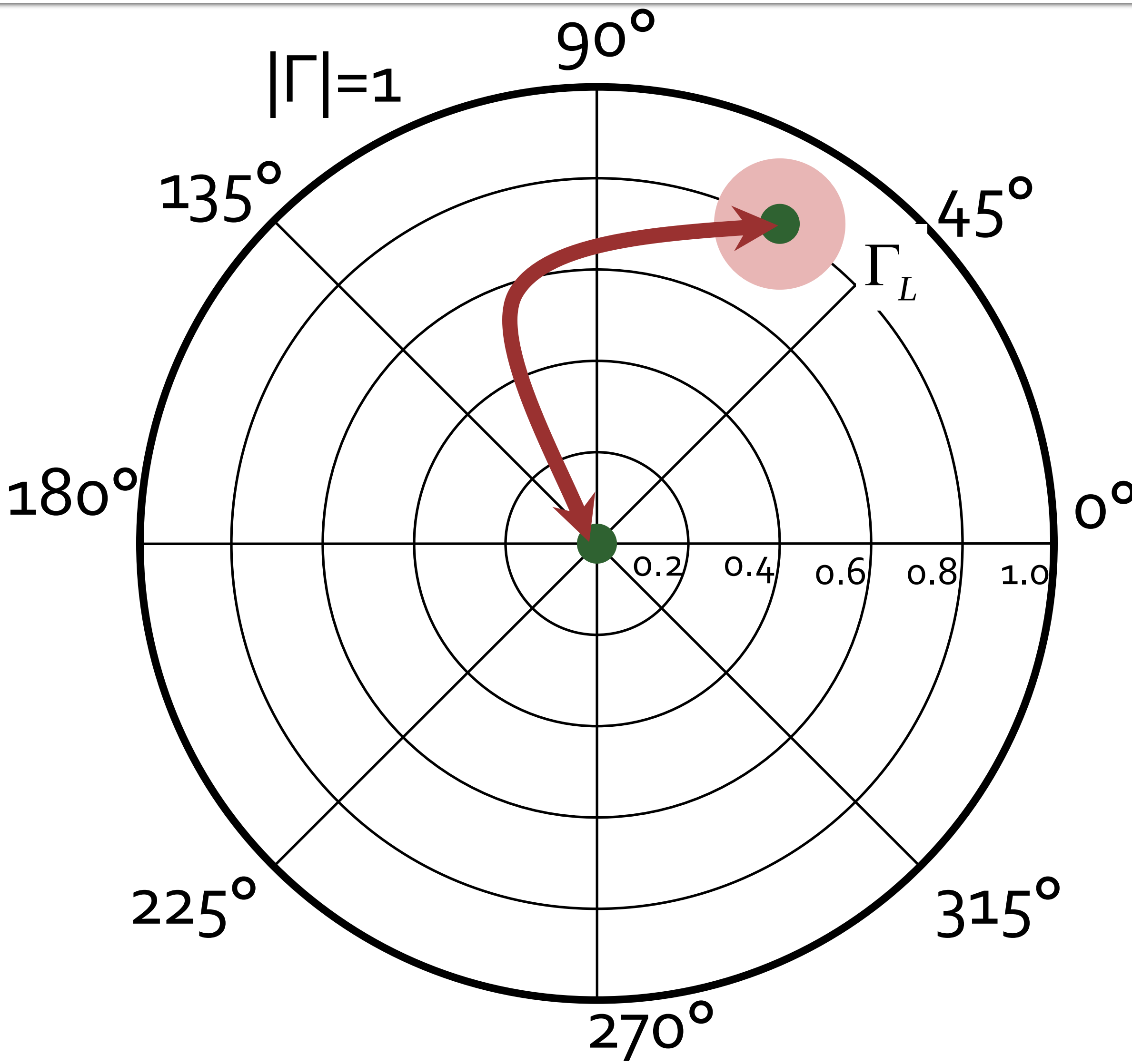
The source (eg. the transistor) having Z_X needs to see a certain reflection coefficient Γ_L towards the load Z_o

The matching circuit must move the point denoting the reflection coefficient in the area where for a Z_o load (Γ_o=0) we see towards it:

$\Gamma = \Gamma_L$ perfect match

$|\Gamma - \Gamma_L| \leq \Gamma_m$ "good enough" match

The Smith Chart, matching , $Z_L \neq Z_o$, $Z_L = Z_o$



- The matching sections needed to move
  - Γ_L in Γ_o
  - Γ_o in Γ_L
- are **identical**. They differ only by the **order** in which the elements are introduced into the matching circuit
- As a result, we can use in match design the same:
  - **methods**
  - **formulae**

Simultaneous matching

$$\Gamma_{in} = \Gamma_S^*$$

$$\Gamma_{in} = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$

$$\Gamma_S^* = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$

$$\Gamma_{out} = \Gamma_L^*$$

$$\Gamma_{out} = S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}$$

$$\Gamma_L^* = S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}$$

- We find Γ_S

$$\Gamma_S = S_{11}^* + \frac{S_{12}^* \cdot S_{21}^*}{1/\Gamma_L^* - S_{22}^*}$$

$$\Gamma_L^* = \frac{S_{22} - \Delta \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}$$

$$\Gamma_S \cdot \left(1 - |S_{22}|^2\right) + \Gamma_S^2 \cdot \left(\Delta \cdot S_{22}^* - S_{11}\right) = \Gamma_S \cdot \left(\Delta \cdot S_{11}^* \cdot S_{22}^* - |S_{22}|^2 - \Delta \cdot S_{12}^* \cdot S_{21}^*\right) + S_{11}^* \cdot \left(1 - |S_{22}|^2\right) + S_{12}^* \cdot S_{21}^* \cdot S_{22}$$

Simultaneous matching

$$\Delta \cdot \left(S_{11}^* \cdot S_{22}^* - S_{12}^* \cdot S_{21}^*\right) = \left|\Delta\right|^2$$

$$\Gamma_S^2 \cdot \left(S_{11} - \Delta \cdot S_{22}^*\right) + \Gamma_S \cdot \left(\left|\Delta\right|^2 - \left|S_{11}\right|^2 + \left|S_{22}\right|^2 - 1\right) + \left(S_{11}^* - \Delta^* \cdot S_{22}\right) = 0$$

- A quadratic equation

$$\Gamma_S = \frac{B_1 \pm \sqrt{B_1^2 - 4 \cdot \left|C_1\right|^2}}{2 \cdot C_1}$$

- Similarly

$$\Gamma_L = \frac{B_2 \pm \sqrt{B_2^2 - 4 \cdot \left|C_2\right|^2}}{2 \cdot C_2}$$

- With variables defined as:

$$\begin{cases} B_1 = 1 + \left|S_{11}\right|^2 - \left|S_{22}\right|^2 - \left|\Delta\right|^2 \\ C_1 = S_{11} - \Delta \cdot S_{22}^* \end{cases} \quad \begin{cases} B_2 = 1 + \left|S_{22}\right|^2 - \left|S_{11}\right|^2 - \left|\Delta\right|^2 \\ C_2 = S_{22} - \Delta \cdot S_{11}^* \end{cases}$$

Simultaneous matching

- Simultaneous matching is possible if:

$$B_1^2 - 4\cdot|C_1|^2 > 0 \qquad B_2^2 - 4\cdot|C_2|^2 > 0$$

$$\Delta\cdot\left(S_{11}^*\cdot S_{22}^* - S_{12}^*\cdot S_{21}^*\right) = |\Delta|^2$$

$$|C_1|^2 = \left|S_{11} - \Delta\cdot S_{22}^*\right|^2 = |S_{12}|^2\cdot|S_{21}|^2 + \left(1-|S_{22}|^2\right)\cdot\left(|S_{11}|^2 - |\Delta|^2\right)$$

$$B_1^2 - 4\cdot|C_1|^2 = \left(1+|S_{11}|^2\right)^2 + \left(|S_{22}|^2 + |\Delta|^2\right)^2 -$$
$$-2\cdot\left(1+|S_{11}|^2\right)\cdot\left(|S_{22}|^2 + |\Delta|^2\right) - 4\cdot|S_{12}\cdot S_{21}|^2 - 4\cdot\left(1-|S_{22}|^2\right)\cdot\left(|S_{22}|^2 - |\Delta|^2\right)$$

$$B_1^2 - 4\cdot|C_1|^2 = \left(1+|S_{11}|^2\right)^2 + \left(|S_{22}|^2 + |\Delta|^2\right)^2 -$$
$$-4\cdot|S_{11}|^2 - 4\cdot|S_{22}|^2\cdot|\Delta|^2 - 2\cdot\left(1-|S_{11}|^2\right)\cdot\left(|S_{22}|^2 - |\Delta|^2\right) - 4\cdot|S_{12}\cdot S_{21}|^2$$

Simultaneous matching

$$B_1^2 - 4\cdot|C_1|^2 = \left(1+|S_{11}|^2\right)^2 + \left(|S_{22}|^2 + |\Delta|^2\right)^2 -$$
$$-4\cdot|S_{11}|^2 - 4\cdot|S_{22}|^2\cdot|\Delta|^2 - 2\cdot\left(1-|S_{11}|^2\right)\cdot\left(|S_{22}|^2 - |\Delta|^2\right) - 4\cdot|S_{12}\cdot S_{21}|^2$$

$$B_1^2 - 4\cdot|C_1|^2 = \left(1-|S_{11}|^2\right)^2 + \left(|S_{22}|^2 - |\Delta|^2\right)^2 - 2\cdot\left(1-|S_{11}|^2\right)\cdot\left(|S_{22}|^2 - |\Delta|^2\right) - 4\cdot|S_{12}\cdot S_{21}|^2$$

$$B_1^2 - 4\cdot|C_1|^2 = \left(1-|S_{11}|^2 - |S_{22}|^2 + |\Delta|^2\right)^2 - 4\cdot|S_{12}\cdot S_{21}|^2$$

$$B_1^2 - 4\cdot|C_1|^2 = \left(K\cdot 2\cdot|S_{12}\cdot S_{21}|\right)^2 - 4\cdot|S_{12}\cdot S_{21}|^2$$

$$B_1^2 - 4\cdot|C_1|^2 = 4\cdot|S_{12}|^2\cdot|S_{21}|^2\cdot\left(K^2-1\right)$$

- Similarly

$$B_2^2 - 4\cdot|C_2|^2 = 4\cdot|S_{12}|^2\cdot|S_{21}|^2\cdot\left(K^2-1\right)$$

Simultaneous matching

$$\Gamma_S = \frac{B_1 \pm \sqrt{B_1^2 - 4\cdot|C_1|^2}}{2\cdot C_1} \qquad \Gamma_L = \frac{B_2 \pm \sqrt{B_2^2 - 4\cdot|C_2|^2}}{2\cdot C_2}$$

- Solutions must ensure stability:

$$|\Gamma_S| < 1 \qquad |\Gamma_L| < 1$$

$$|\Delta| = |S_{11}\cdot S_{22} - S_{12}\cdot S_{21}| < 1 \qquad \begin{cases} B_1 > 0 \\ B_2 > 0 \end{cases}$$

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2\cdot|S_{12}\cdot S_{21}|} > 1 \qquad \begin{cases} B_1^2 - 4\cdot|C_1|^2 = 4\cdot|S_{12}|^2\cdot|S_{21}|^2\cdot\left(K^2-1\right) > 0 \\ B_2^2 - 4\cdot|C_2|^2 = 4\cdot|S_{12}|^2\cdot|S_{21}|^2\cdot\left(K^2-1\right) > 0 \end{cases}$$

Simultaneous matching

- Simultaneous matching can be achieved **if and only if** the amplifier is **unconditionally stable** at the operating frequency, and |Γ|<1 solutions are those with "–" sign of quadratic solutions

$$\Gamma_S = \frac{B_1 - \sqrt{B_1^2 - 4\cdot|C_1|^2}}{2\cdot C_1} \qquad \Gamma_L = \frac{B_2 - \sqrt{B_2^2 - 4\cdot|C_2|^2}}{2\cdot C_2}$$

$$\begin{cases} B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \\ C_1 = S_{11} - \Delta\cdot S_{22}^* \end{cases} \qquad \begin{cases} B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \\ C_2 = S_{22} - \Delta\cdot S_{11}^* \end{cases}$$

Simultaneous matching

- In the case of the simultaneous matching the amplifier achieves the maximum transducer power gain for the bilateral transistor

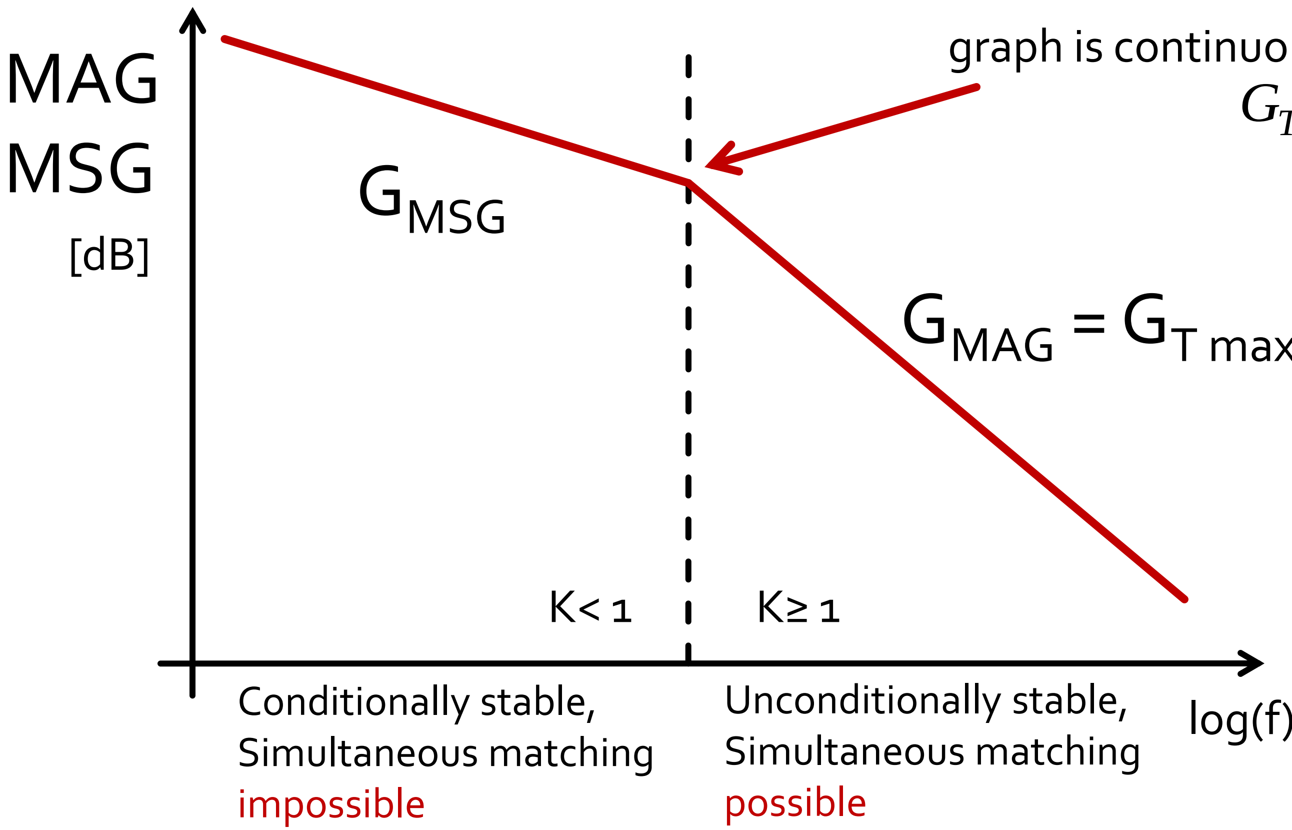
$$G_{T\max} = \frac{|S_{21}|}{|S_{12}|} \cdot \left(K - \sqrt{K^2 - 1}\right)$$

- If the device is **not unconditionaly stable** at a certain frequency we can use MSG (Maximum Stable Gain) as an indicator of the capability to obtain a power gain in stable conditions

$$G_{MSG} = \frac{|S_{21}|}{|S_{12}|}$$

Maximum Available Gain

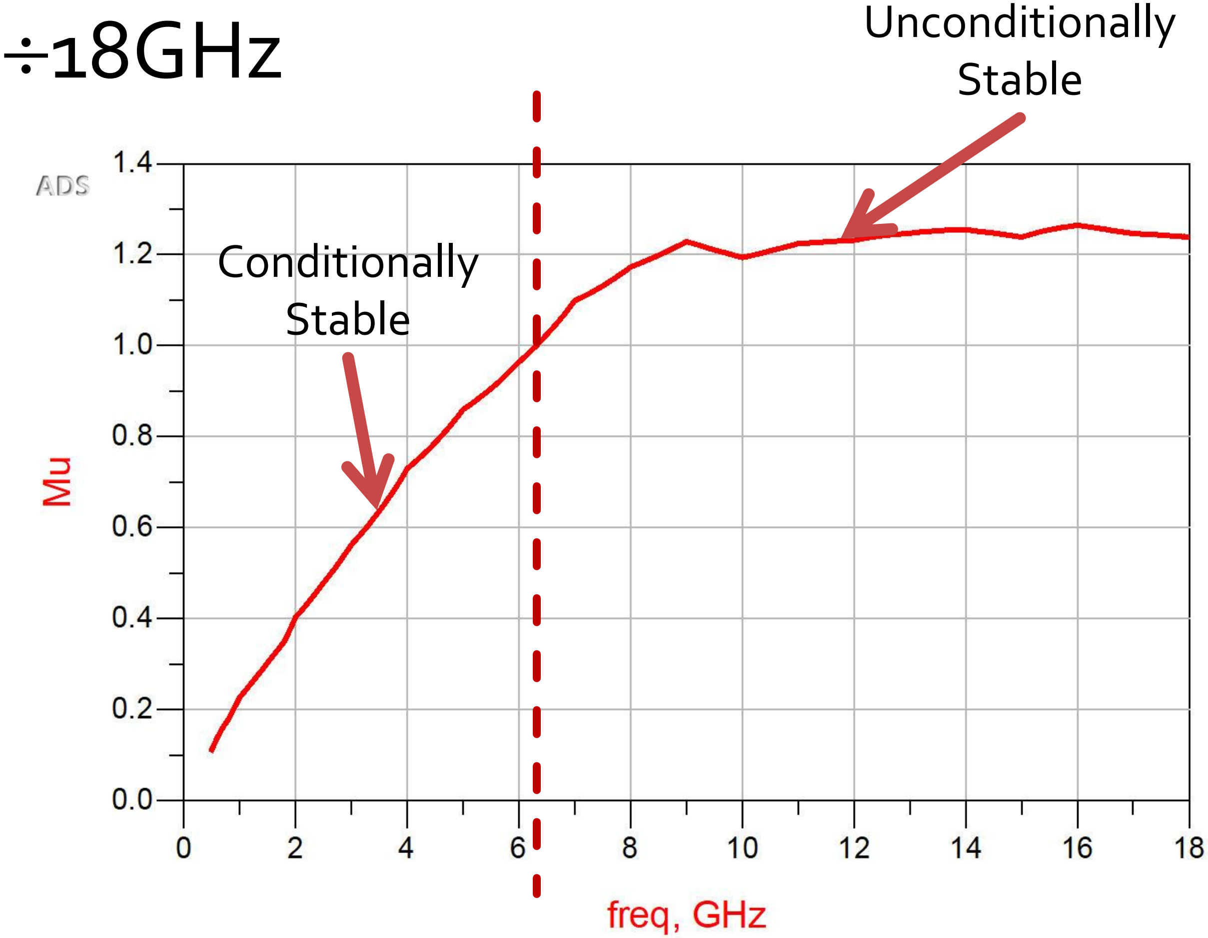
- Indicator across full frequency range of the capability to obtain a power gain



$$G_{T\max}\Big|_{K=1} = \frac{|S_{21}|}{|S_{12}|} = G_{MSG}$$

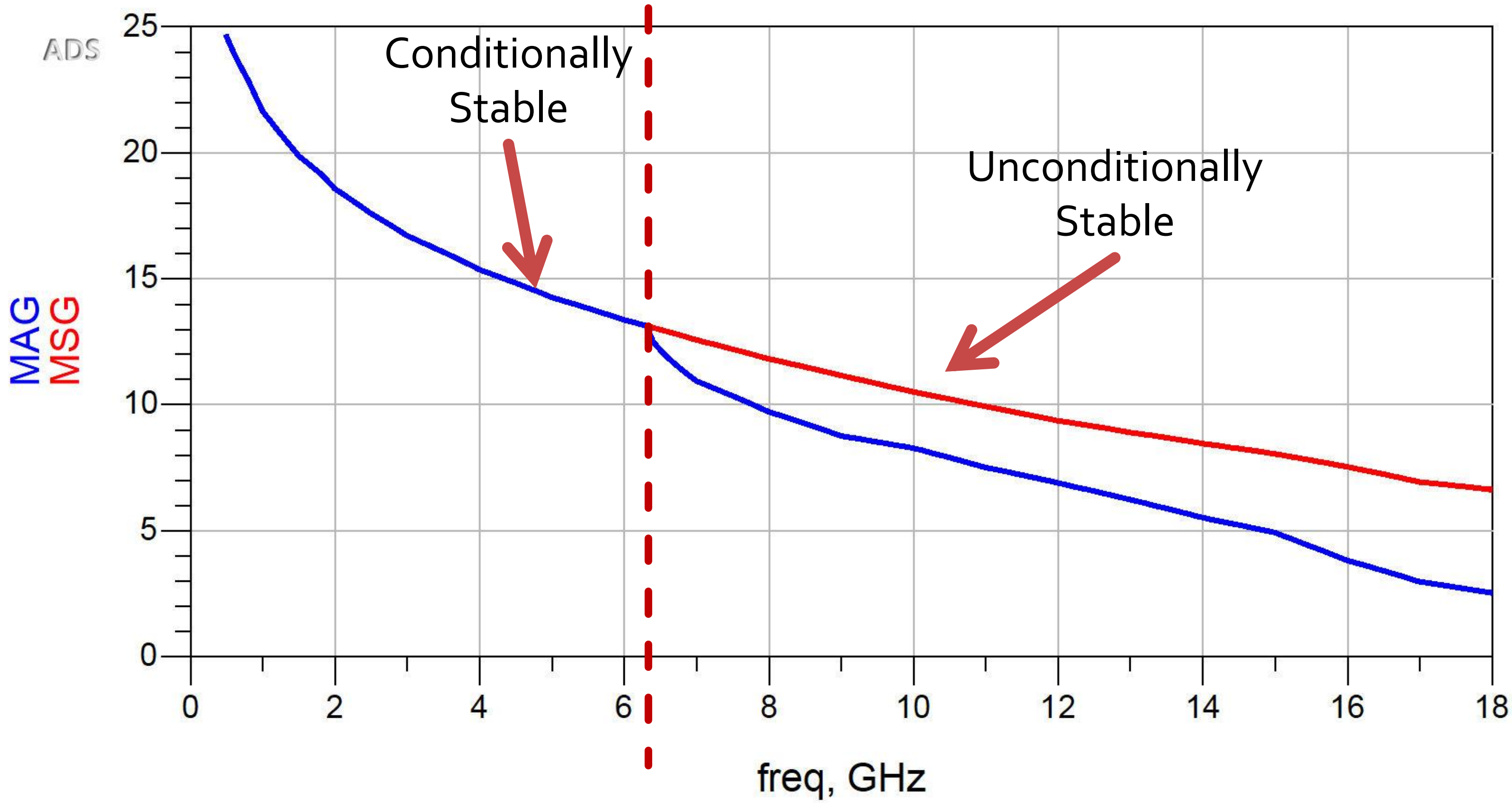
Stability

- ATF-34143 **at Vds=3V Id=20mA**.
- @0.5÷18GHz



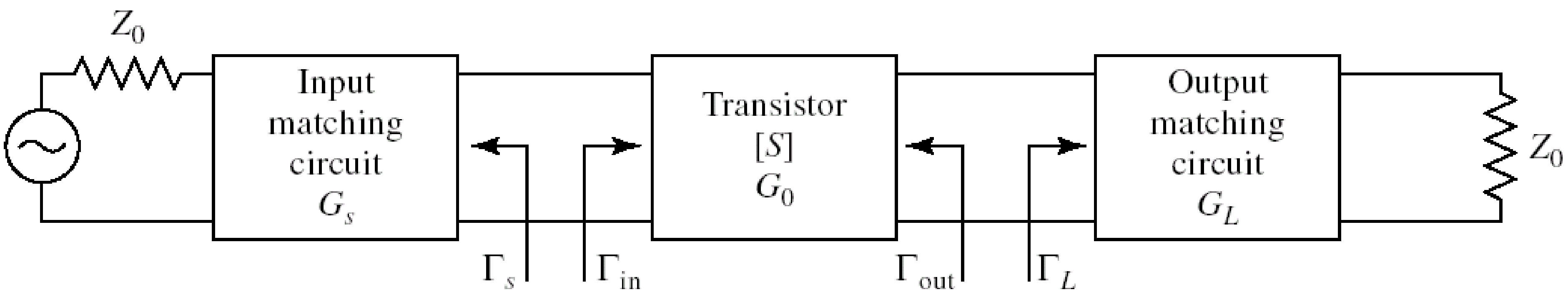
MAG/MSG

- ATF-34143 **at Vds=3V Id=20mA**.
- @0.5÷18GHz



Simultaneous matching, unilateral transistor

- In the case of **unilateral** amplifier/transistor ($S_{12} = 0$) simultaneous matching implies:



$$\Gamma_{in} = S_{11} \qquad \Gamma_{out} = S_{22}$$

$$\Gamma_S = S_{11}^* \qquad \Gamma_L = S_{22}^*$$

$$G_{T\max} = \frac{1}{1-|\Gamma_S|^2} \cdot |S_{21}|^2 \cdot \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2} \qquad G_{TU\max} = \frac{1}{1-|S_{11}|^2} \cdot |S_{21}|^2 \cdot \frac{1}{1-|S_{22}|^2}$$

Example

- ATF-34143 **at Vds=3V Id=20mA**.
  - without stabilization K = **0.886**, MAG = 14.248dB @ 5GHz
  - cannot be used with this bias conditions
- ATF-34143 **at Vds=4V Id=40mA**
  - without stabilization K = **1.031**, MAG = 12.9dB @ 5GHz
  - we use this bias conditions for simultaneous matching

Example

- ATF-34143 **at Vds=4V Id=40mA**.
- @5GHz
  - $S_{11} = 0.64\angle 111°$
  - $S_{12} = 0.117\angle -27°$
  - $S_{21} = 2.923\angle -6°$
  - $S_{22} = 0.21\angle 111°$

Computations

- Complex S Parameters
  - S11 = -0.229+0.597·j
  - S12 = 0.104-0.053·j
  - S21 = 2.907-0.306·j
  - S22 = -0.075+0.196·j

$$G_{T\max} = \frac{|S_{21}|}{|S_{12}|} \cdot \left(K - \sqrt{K^2 - 1}\right) = 19.497 = 12.9\,\text{dB}$$

$$G_{TU\max} = \frac{1}{1-|S_{11}|^2} \cdot |S_{21}|^2 \cdot \frac{1}{1-|S_{22}|^2} = 15.139 = 11.8\,\text{dB}$$

Computations

$$\begin{cases} B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \\ C_1 = S_{11} - \Delta \cdot S_{22}^* \end{cases}$$

$$\begin{cases} B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \\ C_2 = S_{22} - \Delta \cdot S_{11}^* \end{cases}$$

$$\begin{cases} B_1 = ? \\ C_1 = ? \end{cases}$$

$$\begin{cases} B_2 = ? \\ C_2 = ? \end{cases}$$

$$\Gamma_S = \frac{B_1 - \sqrt{B_1^2 - 4 \cdot |C_1|^2}}{2 \cdot C_1}$$

$$\Gamma_L = \frac{B_2 - \sqrt{B_2^2 - 4 \cdot |C_2|^2}}{2 \cdot C_2}$$

$$\Gamma_S = ?$$

$$\Gamma_L = ?$$

Computations

$$\begin{cases} B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \\ C_1 = S_{11} - \Delta \cdot S_{22}^* \end{cases}$$

$$\begin{cases} B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \\ C_2 = S_{22} - \Delta \cdot S_{11}^* \end{cases}$$

$$\begin{cases} B_1 = 1.207 \\ C_1 = -0.277 + j \cdot 0.529 \end{cases}$$

$$\begin{cases} B_2 = 0.476 \\ C_2 = -0.222 - j \cdot 0.013 \end{cases}$$

$$\Gamma_S = \frac{B_1 - \sqrt{B_1^2 - 4 \cdot |C_1|^2}}{2 \cdot C_1}$$

$$\Gamma_L = \frac{B_2 - \sqrt{B_2^2 - 4 \cdot |C_2|^2}}{2 \cdot C_2}$$

$$\Gamma_S = -0.403 - j \cdot 0.768$$

$$|\Gamma_S| = 0.867 < 1$$

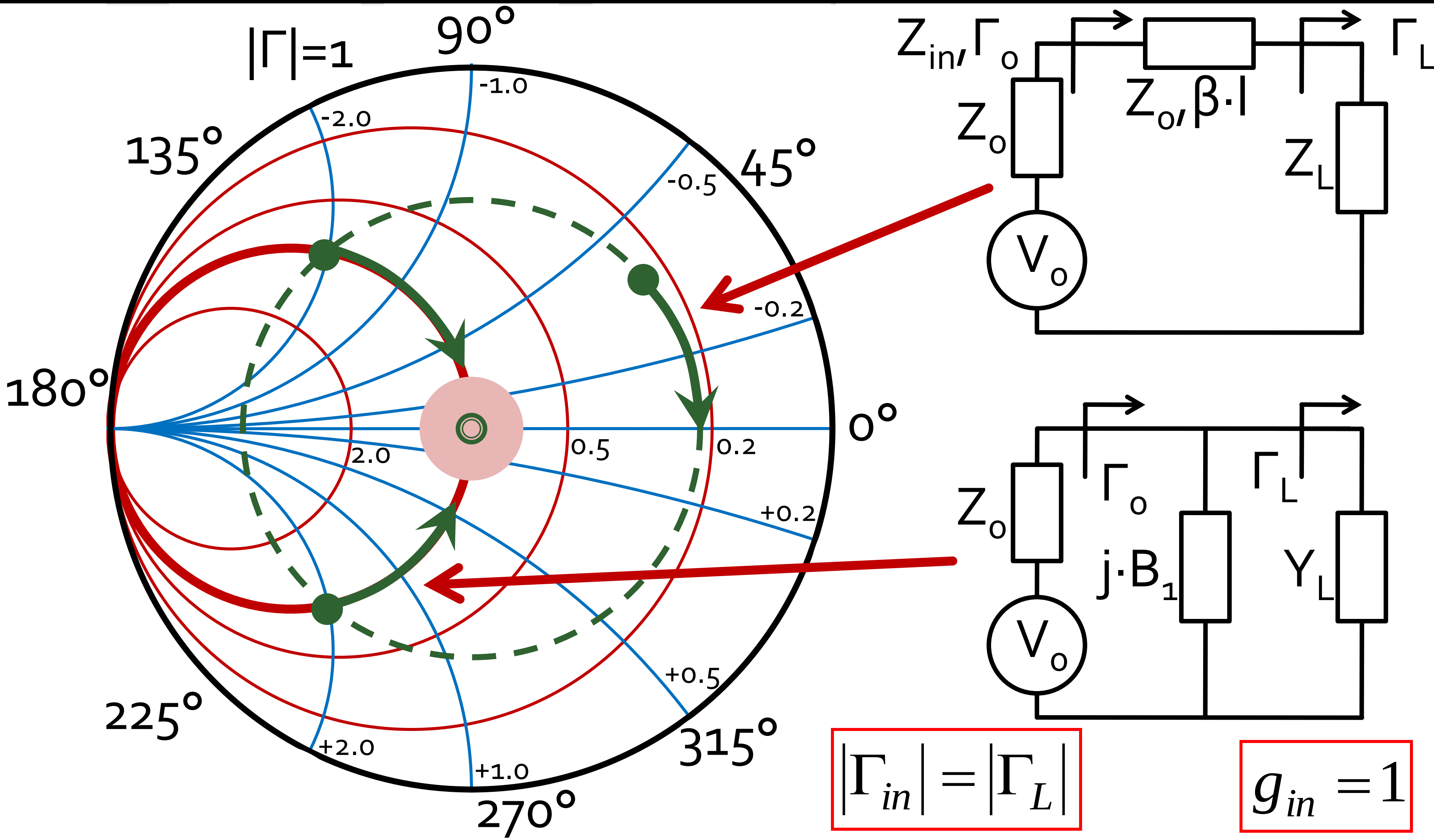
$$\Gamma_S = 0.867 \angle -117.7^\circ$$

$$\Gamma_L = -0.685 + j \cdot 0.04$$

$$|\Gamma_L| = 0.686 < 1$$

$$\Gamma_L = 0.686 \angle 176.7^\circ$$

Shunt stub matching, C6-7



$$|\Gamma_{in}| = |\Gamma_L|$$

$$g_{in} = 1$$

Analytical solution (Γ_S)

$$\cos(\varphi+2\theta)=-|\Gamma_S|$$

$$\theta_{sp}=\beta\cdot l=\tan^{-1}\frac{\mp 2\cdot|\Gamma_S|}{\sqrt{1-|\Gamma_S|^2}}$$

$$\Gamma_S=0.867\angle-117.7°$$

$$|\Gamma_S|=0.867;\quad \varphi=-117.7° \qquad \cos(\varphi+2\theta)=-0.867\Rightarrow\quad(\varphi+2\theta)=\pm150.1°$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation
  - **"+" solution**

    $$(-117.7°+2\theta)=+150.1° \qquad \theta=133.9° \qquad \operatorname{Im} y_S=\frac{-2\cdot|\Gamma_S|}{\sqrt{1-|\Gamma_S|^2}}=-3.477$$

    $$\theta_{sp}=\tan^{-1}(\operatorname{Im} y_S)=-74°(+180°)\rightarrow\theta_{sp}=106°$$

  - **"-" solution**

    $$(-117.7°+2\theta)=-150.1° \qquad \theta=-16.2°(+180°)\rightarrow\theta=163.8°$$

    $$\operatorname{Im} y_S=\frac{+2\cdot|\Gamma_S|}{\sqrt{1-|\Gamma_S|^2}}=+3.477 \qquad \theta_{sp}=\tan^{-1}(\operatorname{Im} y_S)=74°$$

Analytical solution (Γ_L)

$$\cos(\varphi + 2\theta) = -|\Gamma_L|$$

$$\theta_{sp} = \beta \cdot l = \tan^{-1} \frac{\mp 2 \cdot |\Gamma_L|}{\sqrt{1 - |\Gamma_L|^2}}$$

$$\Gamma_L = 0.686 \angle 176.7°$$

$$|\Gamma_L| = 0.686; \quad \varphi = 176.7°$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation
  - **"+" solution**
  - **"-" solution**

Analytical solution (Γ_L)

$$\cos(\varphi + 2\theta) = -|\Gamma_L|$$

$$\theta_{sp} = \beta \cdot l = \tan^{-1} \frac{\mp 2 \cdot |\Gamma_L|}{\sqrt{1 - |\Gamma_L|^2}}$$

$$\Gamma_L = 0.686\angle 176.7°$$

$$|\Gamma_L| = 0.686; \quad \varphi = 176.7° \qquad \cos(\varphi + 2\theta) = -0.686 \Rightarrow \quad (\varphi + 2\theta) = \pm 133.3°$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation
  - **"+" solution**

$$(176.7° + 2\theta) = +133.3° \qquad \theta = -21.7°(+180°) \to \theta = 158.3°$$

$$\theta_{sp} = \tan^{-1}(\operatorname{Im} y_L) = -62.1°(+180°) \to \theta_{sp} = 117.9° \qquad \operatorname{Im} y_L = \frac{-2 \cdot |\Gamma_L|}{\sqrt{1 - |\Gamma_L|^2}} = -1.885$$

  - **"-" solution**

$$(176.7° + 2\theta) = -133.3° \qquad \theta = -155°(+180°) \to \theta = 25°$$

$$\operatorname{Im} y_L = \frac{+2 \cdot |\Gamma_L|}{\sqrt{1 - |\Gamma_L|^2}} = +1.885 \qquad \theta_{sp} = \tan^{-1}(\operatorname{Im} y_L) = 62.1°$$

Complete analytical solution

- We choose **one** of the two possible solutions for the input matching

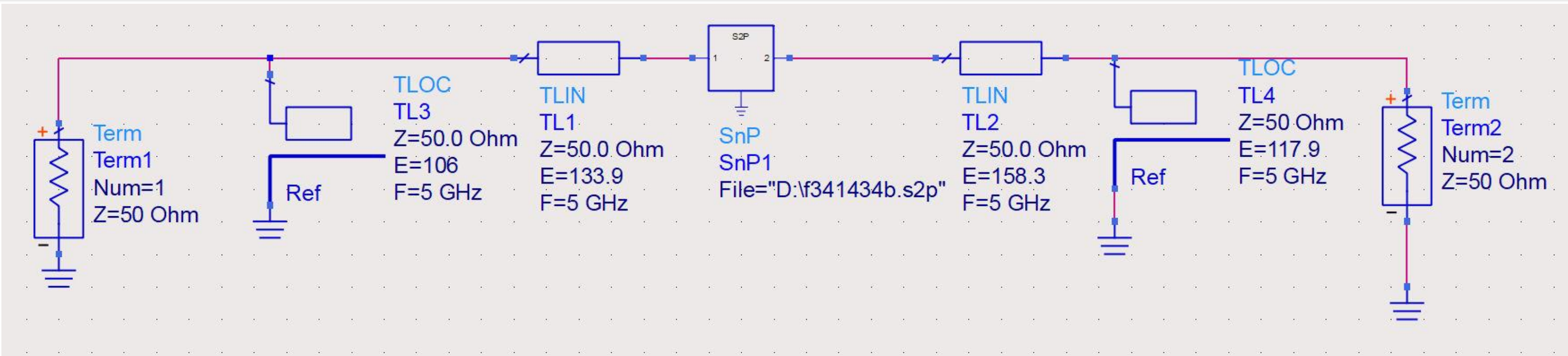
$$(\varphi+2\theta)=\begin{cases}+150.1^\circ\\-150.1^\circ\end{cases}\quad \theta=\begin{cases}133.9^\circ\\163.8^\circ\end{cases}\quad \mathrm{Im}[y_S(\theta)]=\begin{cases}-3.477\\+3.477\end{cases}\quad \theta_{sp}=\begin{cases}-74^\circ+180^\circ=106^\circ\\+74^\circ\end{cases}$$

- Similarly for the output matching

$$(\varphi+2\theta)=\begin{cases}+133.3^\circ\\-133.3^\circ\end{cases}\quad \theta=\begin{cases}158.3^\circ\\25.0^\circ\end{cases}\quad \mathrm{Im}[y_S(\theta)]=\begin{cases}-1.885\\+1.885\end{cases}\quad \theta_{sp}=\begin{cases}117.9^\circ\\62.1^\circ\end{cases}$$

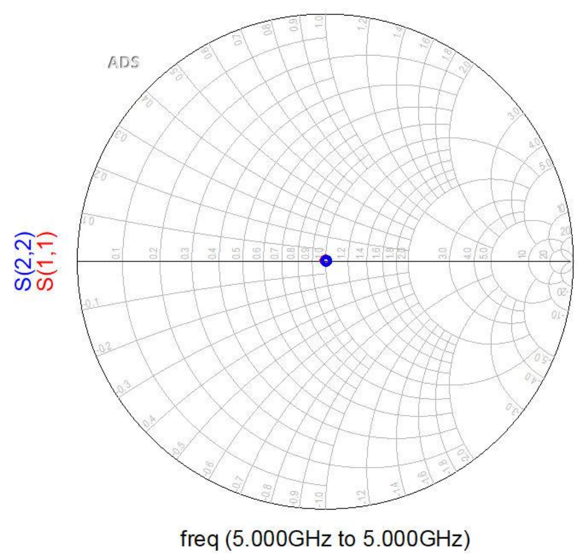
- In total there are **4** possible solutions input/output

ADS

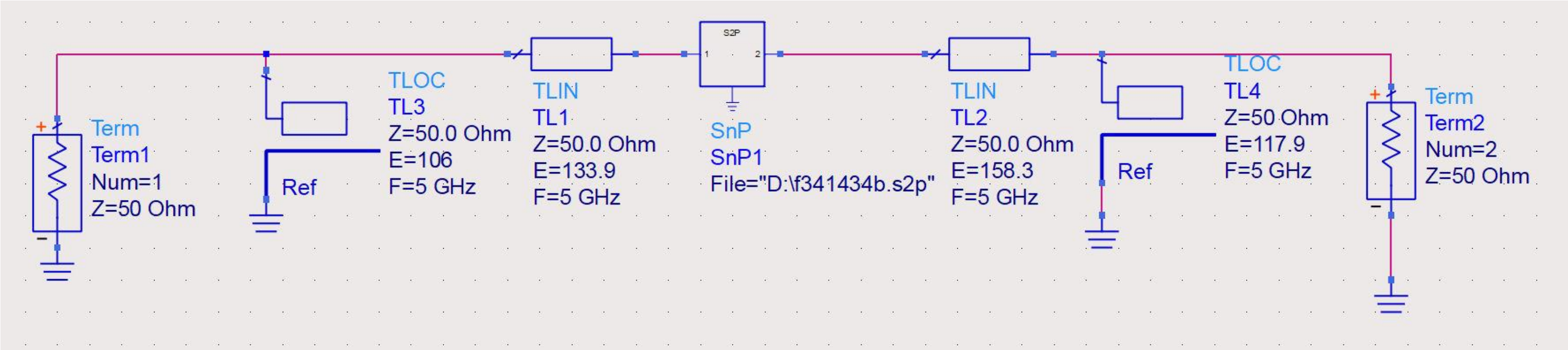


Eqn GT=10*log(mag(S(2,1))**2)

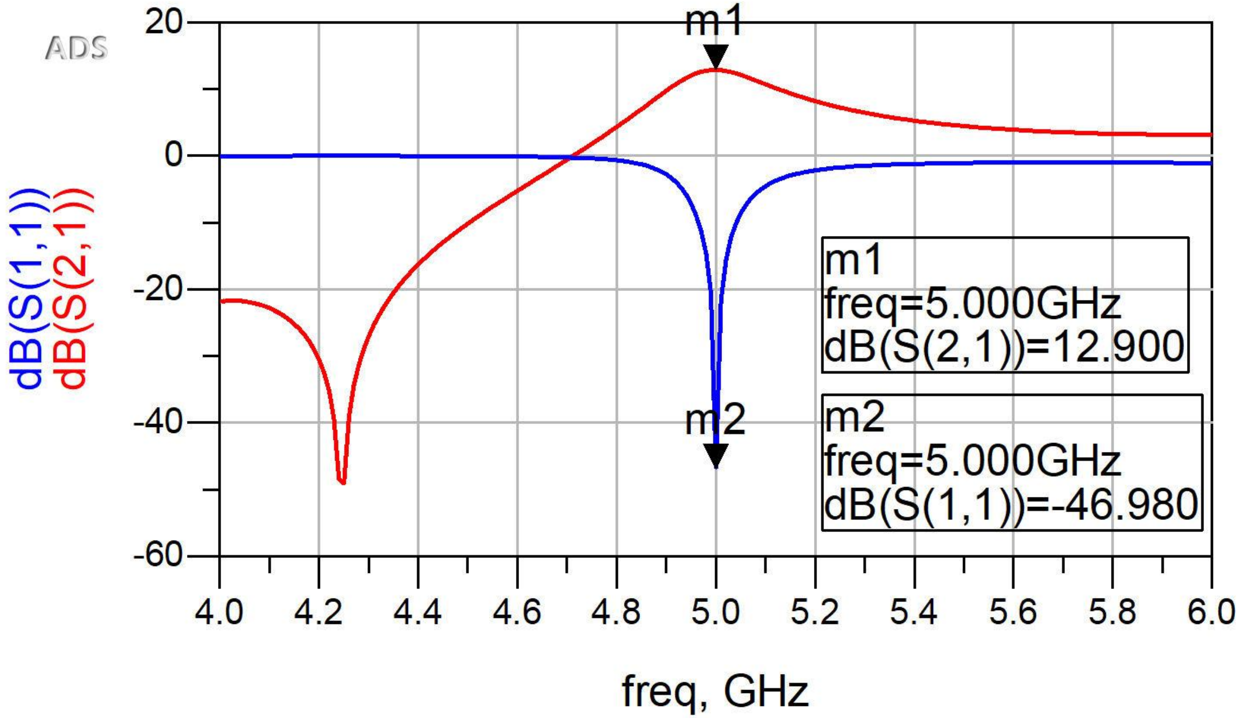
freq	S(2,1)	GT	S(1,1)	S(2,2)
5.000 GHz	4.415 / 157.353	12.900	0.004 / 86.088	0.004 / 37.766



ADS



Term
Term1
Num=1
Z=50 Ohm
Ref
TLOC
TL3
Z=50.0 Ohm
E=106
F=5 GHz
TLIN
TL1
Z=50.0 Ohm
E=133.9
F=5 GHz
S2P
SnP
SnP1
File="D:\f341434b.s2p"
TLIN
TL2
Z=50.0 Ohm
E=158.3
F=5 GHz
Ref
TLOC
TL4
Z=50 Ohm
E=117.9
F=5 GHz
Term
Term2
Num=2
Z=50 Ohm



ADS
dB(S(1,1))
dB(S(2,1))
20
0
-20
-40
-60
m1
m2
m1
freq=5.000GHz
dB(S(2,1))=12.900
m2
freq=5.000GHz
dB(S(1,1))=-46.980
4.0
4.2
4.4
4.6
4.8
5.0
5.2
5.4
5.6
5.8
6.0
freq, GHz

Contact

- Microwave and Optoelectronics Laboratory
- **http://rf-opto.etti.tuiasi.ro**
- **rdamian@etti.tuiasi.ro**